PHILOSOPHICAL TRANSACTIONS.

PART II.

PHILOSOPHICAL TRANSACTIONS,

OF THE

ROYAL SOCIETY

OF

LONDON.

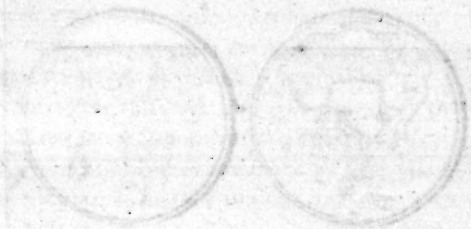
VOL. LXX. For the Year 1780.

PART II.

LONDON,

SOLD BY LOCKYER DAVIS, AND PETER ELMSLY,
PRINTERS TO THE ROYAL SOCIETY.

MDCCLXXXI.



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PHILOSOPHICAL TRANSACTIONS.

XVII. *Theorems for computing Logarithms.* By the Rev. John Hellins; *communicated by the Rev.* Nevil Maſkelyne, *D. D. F. R. S. and Aſtronomer Royal.*

Read March 16, 1780.

THE utility of logarithms is ſo well known, that much need not be ſaid upon it. In our days he muſt be a ſlender mathematician who does not know that they are uſeful, not only in trigonometry, navigation, aſtronomy, the calculations of compound intereſt

and annuities, but alſo in the finding of fluents, and the ſummation of infinite ſeries.

Some of the greateſt mathematicians that this kingdom ever produced, as Sir ISAAC NEWTON, Dr. HALLEY, Mr. COTES, and Mr. SIMPSON, have thought it not beneath them to improve the conſtruction of logarithms, which ſtrongly argues the utility of thoſe artificial numbers, and may ſuggeſt to us that the conſtruction of them cannot be much further improved.

Now, although we ſhould be very diffident in our expectations of improvement in any part of the mathematics after it has been handled by ſuch great men, yet, if the method of computing be in general long and tedious, or if there ſtill remain any particular difficulty, I believe, no good reaſon can be given why every attempt to abridge the one, or remove the other, ſhould be diſcouraged. The eaſy method of computing the logarithms of large numbers given in page 49. of Mr. SIMPSON's pamphlet on Trigonometry is a proof that thoſe gentlemen, who were of opinion that nothing better was to be hoped for, or expected, than what they publiſhed on the ſubject in the beginning of this century, were miſtaken. And the following theorems, inferior to none as to convergency, and uſeful in deducing the logarithms of great fractions from thoſe of ſmall ones, or

the

the logarithms of ſmall numbers from thoſe of great ones, may be conſidered as another proof of the miſtake before mentioned. I have only to add here, that theſe theorems are new to me; and if they are ſo to the public, I humbly preſume they will be acceptable.

THEOREM I.

The log. of $\frac{p+q}{p}$ = 2 × log. of $\frac{2p+2q}{2p+q}$ + log. of $\frac{\overline{2p+q}^2}{\overline{2p+q}^2-qq}$.

DEMONSTRATION.

$\frac{\overline{2p+q}^2}{\overline{2p+q}^2-qq} \times \frac{\overline{2p+2q}^2}{\overline{2p+q}^2} = \frac{1}{\overline{2p+q}^2-qq} \times \frac{\overline{2p+2q}^2}{1} = \frac{4pp+8pq+4qq}{4pp+4pq} = \frac{pp+2pq+qq}{p \times p+q} = \frac{p+q}{p}$: conſequently, log. $\frac{\overline{2p+q}^2}{\overline{2p+q}^2-qq}$ + 2 l. $\frac{2p+2q}{2p+q}$ = log. $\frac{p+q}{p}$. Q. E. D.

COROLLARY.

If $q=1$, and we write n for p, the theorem becomes log. $\frac{n+1}{n}$ = 2 log. $\frac{2n+2}{2n+1}$ + log. $\frac{\overline{2n+1}^2}{\overline{2n+1}^2-1}$, which expreſſion perhaps is of more frequent uſe than that above.

THEOREM II.

Log. $\frac{p+q}{p}$ = 2 log. $\frac{2p+q}{2p}$ − log. $\frac{\overline{2p+q}^2}{\overline{2p+q}^2-qq}$.

DEMONSTRATION.

$\frac{2p+2q}{2p+q} \times \frac{2p+q}{2p} = \frac{2p+2q}{2p} = \frac{p+q}{p}$; therefore, log. $\frac{p+q}{p}$ = log. $\frac{2p+2q}{2p+q}$ + log. $\frac{2p+q}{2p}$. But it has been proved above, that log. $\frac{p+q}{p}$ = 2 log. $\frac{2p+2q}{2p+q}$ + log. $\frac{\overline{2p+q}^2}{\overline{2p+q}^2-qq}$. If now we take this equation from twice the laſt there will remain 2 log. $\frac{p+q}{p}$ − log. $\frac{p+q}{p}$ = 2 log. $\frac{2p+2q}{2p+q}$ + 2 log. $\frac{2p+q}{2p}$ − 2 log. $\frac{2p+2q}{2p+q}$ − log. $\frac{\overline{2p+q}^2}{\overline{2p+q}^2-qq}$: that is, log. $\frac{p+q}{p}$ = 2 log. $\frac{2p+q}{2p}$ − log. $\frac{\overline{2p+q}^2}{\overline{2p+q}^2-qq}$. Q. E. D.

COROLLARY.

Putting $q=1$, and $n=p$, as above, we have

$$\log. \frac{n+1}{n} = 2 \log. \frac{2n+1}{2n} - \log. \frac{\overline{2n+1}^2}{\overline{2n+1}^2-1}.$$

I ſhall now ſet down ſome examples of the uſe of theſe theorems beginning with theorem 1.

The firſt example of the utility of this theorem may be in computing the logarithm of the number 2.

It is well known to mathematicians, that the computation of this logarithm was formerly a very laborious

taſk: and although the work be much ſhortened by help of the converging ſeries invented by the illuſtrious Sir ISAAC NEWTON, ſtill the logarithm of 2 has not been directly computed without many figures by any theorem I have yet ſeen. The eaſieſt computation of it that has come to my hands is in page 44. of the late ingenious Mr. THOMAS SIMPSON's pamphlet on Trigonometry and logarithms. His ſeries conſiſts of the powers of $\frac{1}{3}$.

If now we put $n = 1$ in the theorem ($\log. \frac{n+1}{n} = 2 \log. \frac{2n+2}{2n+1} + \log. \frac{\overline{2n+1}^2}{\overline{2n+1}^2 - 1}$) we ſhall have $\log. \frac{2}{1} = 2 \log. \frac{4}{3} + \log. \frac{9}{8}$. Here then the fractions, whoſe odd powers are to be uſed, are $\frac{1}{7}$ and $\frac{1}{17}$; conſequently, in the ſeries formed from $\frac{1}{7}$, about one half of the number of terms taken by Mr. SIMPSON will give the reſult true to as many places of figures as his; and, from the fraction $\frac{1}{17}$, much fewer terms will ſuffice. To ſhow how faſt theſe ſeries converge I will ſet down of each terms, enough to give the logarithm of 2 true to ten places of figures.

The

The odd powers of $\frac{1}{3}$ divided by their respective indices.

1st,	0·142857142 86
3d,	0·000971817 30
5th,	0·000011899 80
7th,	0·000000173 47
9th,	0·000000002 75
11th,	0·000000000 04
The sum,	0·143841036 22 is $\frac{1}{4}$ l. of $\frac{3}{2}$.
	4
	0·575364148 8 twice l. $\frac{3}{2}$.

The odd powers of $\frac{1}{17}$ divided by their respective indices.

1st,	0·058823529 41
3d,	0·000067847 21
5th,	0·000000140 86
7th,	0·000000000 35
The sum,	0·058891517 83 is $\frac{1}{2}$ l. of $\frac{9}{8}$.
	2
Log. $\frac{9}{8}$,	0·117783035 66
2 log. $\frac{3}{2}$,	0·575364144 88
Log. of 2.	0·693147180 54

But it is obvious, that this operation gives not only the logarithm of 2 but that of 3 alſo: for the logarithm of 4 being given from that of 2, and the logarithm of $\frac{4}{3}$ computed above, the logarithm of 3 is had, being = log. of 4 − log. of $\frac{4}{3}$.

Log. of 4	1·386294361 08
Log. of $\frac{4}{3}$	0·287682072 44
Log. of 3	1·098612288 64

Other examples of the uſe of theſe theorems in ſhewing how eaſily the logarithms of great fractions are derived from thoſe of ſmall ones.

If the logarithm of $\frac{64}{63}$ were given, or computed, we may very eaſily find the logarithm of $\frac{32}{31}$: for (by the-

orem

orem 1.) $2 \log. \frac{64}{63} + \log. \frac{\overline{63}^2}{63^2 - 1} = \log. \frac{32}{31}$. Here the fraction, whoſe odd powers are to be uſed in the ſeries, is $\frac{1}{7937}$, and the very firſt term of it, will give the logarithm true to twelve places of figures.

Again, if the logarithm of $\frac{16}{15}$ were to be computed from that of $\frac{32}{31}$ found above, we ſhould have $2 \log. \frac{32}{31} + \log. \frac{\overline{31}^2}{31^2 - 1} = \log. \frac{16}{15}$. Here the fraction to be uſed in the ſeries is $\frac{1}{1921}$, the firſt term of which will give the logarithm true to ten places of figures.

In like manner, from the logarithm of $\frac{16}{15}$ we may find that of $\frac{8}{7}$; from logarithm of $\frac{8}{7}$ that of $\frac{4}{3}$; and from the logarithm of $\frac{4}{3}$ that of $\frac{2}{1}$, as is done above. The reſpective fractions for the ſeries will be $\frac{1}{449}$, $\frac{1}{97}$, and $\frac{1}{17}$.

Thus far the fractions I have taken have even numbers for their numerators; let us now take one whoſe numerator is an odd number $\frac{9}{8}$. Here n being $= 3\frac{1}{17}$, $\log. \frac{9}{7} \left(\frac{\frac{16}{15}}{\frac{64}{63}}\right) = 2 \log. \frac{9}{8} + \log. \frac{64}{63}$; and the fraction whoſe odd powers are to be uſed is $\frac{1}{287}$. Hence we have the log. of $\frac{8}{7}$ (for $\frac{9}{7} \div \frac{9}{8} = \frac{8}{7}$) and may proceed to find the logarithm of 2 as above. But the logarithm of $\frac{8}{7}$ may be directly derived from the equation thus: the equation in other terms is, $\log. 9 - \log. 7 = 2 \log. 9 - 2 \log. 8 + \log. \frac{64}{63}$; then, by tranſpoſition, $\log. 8 - \log. 7 = \log. 9 - \log. 8 + \log. \frac{64}{63}$, or $\log. \frac{8}{7} = 2 \log. \frac{9}{8} + \log. \frac{64}{63}$.

But when the numerator of the fraction, whose logarithm is given, is odd, theorem 2. is more commodious. For taking $\frac{9}{8}$, as before, we have $2 \log. \frac{9}{8} - \log. \frac{81}{80} = \log. \frac{5}{4}$, where the fraction to be involved is $\frac{1}{161}$. Again, $2 \log. \frac{5}{4} - \log. \frac{25}{24} = \log. \frac{3}{2}$, where the fraction is $\frac{1}{49}$. And $2 \log. \frac{3}{2} - \log. \frac{9}{8} = \log. \frac{2}{1}$, where we have only to take the difference of logarithms, as the logarithm of $\frac{9}{8}$ as well as that of $\frac{3}{2}$ is given.

All the above calculations are of hyperbolic logarithms; but the same theorems hold good for Mr. BRIGGS's, or any other. I will give an example in the computation of BRIGGS's logarithm of 7 from others already known.

Let the logarithms of 100, 99, and 50, be given; then (by theorem 1.) $2 \log. \frac{100}{99} + \frac{99^2}{99^2-1} = \log. \frac{50}{49}$, or $\log. \frac{100}{99} + \frac{1}{2} \log. \frac{99^2}{99^2-1} = \frac{1}{2} \log. \frac{50}{49}$; and then $\frac{1}{2} \log. 50 + \frac{1}{2} \log. \frac{50}{49} = \frac{1}{1} \log. 49 = \log. 7$.

Log. of $\frac{100}{99}$ - - -	0·004364805402 45
$\frac{1}{2}$ log. of $\frac{99^2}{99^2-1}$ $\left(= \frac{0{\cdot}43429, \&c.}{19601} = \right)$	0·000022156751 28
$\frac{1}{2}$ log. of $\frac{50}{49}$ - - -	0·004386962153 73
$\frac{1}{2}$ log. of 50 - - -	0·849485002168 01
Log. of 7 - - -	0·845098040014 28

SCHOLIUM.

SCHOLIUM.

Neither the number 2, nor the fraction $\frac{64}{63}$ is chosen as the most advantageous to begin with in computing a table of logarithms; but they are taken as some of the first that occurred, to show the use of these theorems. Perhaps there are other instances in which they would be shown to much more advantage; but I hope their use will appear from the few examples given. They may indeed be transformed so as to be more commodious in particular cases, and there may be some others derived from them, one or two of which I will here put down.

It is evident from theorem 1. and 2. that $2 \log. \frac{2p+2q}{2p+q} + \log. \frac{\overline{2p+q}^2}{\overline{2p+q}^2-qq} = 2 \log. \frac{2p+q}{2p} - \log. \frac{\overline{2p+q}^2}{\overline{2p+q}^2-qq}$; consequently, $2 \log. \frac{2p+2q}{2p+q} + 2 \log. \frac{\overline{2p+q}^2}{\overline{2p+q}^2-qq} = 2 \log. \frac{2p+q}{2p}$, or, $\log. \frac{2p+q}{2p} = \log. \frac{2p+2q}{2p+q} + \log. \frac{\overline{2p+q}^2}{\overline{2p+q}^2-qq}$, which may be called theorem 3.

Again, this equation may be thus expressed: $\log. \overline{2p+q} - \log. 2p = \log. \overline{2p+2q} - \log. \overline{2p+q} + \log. \frac{\overline{2p+q}^2}{\overline{2p+q}^2-qq}$; and, by transposition, $2 \log. \overline{2p+q} = \log. \overline{2p+2q} + \log. 2p + \log. \frac{\overline{2p+q}^2}{\overline{2p+q}^2-qq}$, which may be called theorem 4. And this is, in effect, one of the theorems given by Dr. HALLEY,

LEY, in the Philoſophical Tranſactions, N° 216, and of which the doctor ſaid, it converges ſo very faſt, that, in his opinion, nothing better was to be hoped.

There are yet ſome contrivances different from thoſe mentioned in the beginning of SHERWIN's book of mathematical tables, or any other that have come to my hands, whereby the labour of computing a table of logarithms is ſhortened; but to explain them would require more time than my preſent ſituation affords me.

The obſervations and reaſonings which led me to the diſcovery of the above theorems, I imagine, need not here be mentioned. Such as they are, I beg leave to lay them before the candid and ſkilful in theſe matters, in hopes that the invention will appear to them, as it does to me, a uſeful one.

It has, indeed, been objected, by a gentleman of my acquaintance, that improvements in the conſtruction of logarithms cannot now be uſeful, becauſe logarithms are already conſtructed.

I anſwer, that argument, if it has any weight, operates equally againſt Sir ISAAC NEWTON, Dr. HALLEY, Mr. COTES, Mr. SIMPSON, and ſeveral other ingenious mathematicians; for logarithms were invented, and tables of them conſtructed, before their time; ſo that if I ſhould be thought to have miſemployed my time in improving

improving the computation of these artificial numbers, I have some consolation in thinking that I have therein followed the example of the very respectable company just mentioned.

I trust, however, that, with mathematicians, every improvement in calculation will be acceptable.

XVII. *Connoiſſances eſſentielles pour juger de quelque Eſpéce nouvelle de Moulin à Cannes qu'on puiſſe propoſer. Par Monſieur* Cazaud, *Membre de la Societé Royale.*

Read March 16, 1780.

SI l'on exigeoit d'un homme qui a une maſſe conſiderable à déplacer, qu'il en connut exactement le poids, avant d'y appliquer le levier, on exigeroit ſouvent une choſe impoſſible; mais la maſſe une fois ſoulevée avec le dixieme levier, ſi les neuf premiers n'ont pas réuſſi, on ſçait la reſiſtance qu'il faudra deſormais lui oppoſer.

Les premiers moulins à ſucre ont été faits ſans principes; cela devoit être: il eſt vray qu'on y a fait peu ou point de changements eſſentiels; on en propoſe tous les jours qu'on regarde comme tels, et qui dans la pratique ſe reduiſent à peu de choſe. Je crois pouvoir dire ſans indiſcretion que le peu de mecaniciens que j'ay vu diriger leurs vues vers cet objet, n'ont pu me répondre lorſque je leur ai demandé à combien ils evaluoient la reſiſtance des cannes: j'avouerai avec la même franchiſe que

j'avois

j'avois fait moi-même pluſieurs expériences très coûteuſes et fort inutiles, ſans avoir pris la peine de repondre à ma queſtion, dont je ne ſentois que machinalement l'importance: ſur quel principe les autres agiſſoient-ils? ſur quel principe avois-je moi-même agi?

On conſtruiſit à Londres il y a onze ans une machine à feu deſtinée auſſi à preſſer les cannes: l'auteur m'en parla; je lui fis ma queſtion ordinaire, il me repondit qu'il evaluoit la reſiſtance à ſix milliers: ſa réponſe qui préſentoit une idée préciſe m'autoriſa à le prier de me communiquer les fondemens de cette evaluation; il me repondit qu'il etoit impoſſible que la reſiſtance fut plus conſidérable qu'il ne l'avoit ſuppoſée; je le priai de me permettre de refléchir ſur une idée qui me vint dans ce moment là, et qui me paroiſſoit meriter d'être approfondie. La voici; j'etablirai des faits, il ſera facile d'apprecier mes conſequences.

On connoît le mecaniſme de nos moulins à beſtiaux, ils ont ordinairement de 45 à 55 pieds de diamettre, ſuppoſons 50. On applique deux mulets à environ un pied de chacune des extremités d'un bras qui traverſe le grand role auquel eſt attaché le cylindre du milieu; ce cylindre a 17 ou 18 pouces de diamettre; la reſiſtance des cannes ſe trouve donc à neuf pouces du centre de l'action; il faut pour la vaincre un effort continu de quatre mulets

appliqués

appliqués à un levier d'environ 24 pieds; cet effort equivaut à 600 livres, à raison de 150 par mulet. Dans 24 pieds il y a 32 fois neuf pouces, ou 32 fois 600 livres; donc la resistance des cannes est d'environ 19 milliers dans un moulin à bestiaux; car il faut essentiellement remarquer qu'après une demie heure de travail les quatre mulets sont en eau, et qu'on ne les change que de deux heures en deux heures.

Après cet exposé l'auteur de la machine à feu me dit qu'il pouvoit facilement tripler, quadrupler même, sa puissance si cela etoit necessaire; il ne me demanda point d'autres details, qui m'auroient sans doute suggéré dès alors quelques idées qui me sont venues depuis; à l'inspection d'un autre moulin proposé je perdis de vue la machine à feu; deux ans après on me dit qu'on l'avoit envoyée à la Jamaique, j'en ignore le succès, que la renommée auroit probablement fait connoitre si elle eut repondu aux idées de l'auteur.

Voici quelques autres observations qu'il faudroit joindre à la connoissance preliminaire et, je crois, essentielle que je viens de donner.

S'il ne s'agissoit que de vaincre (n'importe en combien de tems) la resistance de 19 milliers dont j'ay parlé, on conçoit qu'en appliquant l'effort continu de deux hommes qu'on évalue à 50 livres, au bout d'un levier de 388

pieds, on auroit un produit egal à celui des quatre mulets du moulin ordinaire, mais alors on n'obtiendroit qu'en douze heures, ce qu'on obtient dans une ſeule avec les quatre mulets.

Il faut ſçavoir auſſi que ce moulin à beſtiaux, au quel on eſt obligé de conſacrer au moins 36 mulets, ne donne cependant, une heure dans l'autre, qu'environ 80 à 100 gallons de liqueur, et qu'un bon moulin à l'eau, tel qu'il le faudroit pour faire ce qu'on appelle rondement 250 à 300 bariques de ſucre dans la bonne ſaiſon, doit donner 160 à 200 gallons de liqueur par heure, l'une dans l'autre.

Il faut ſçavoir auſſi que pour donner ces 160 gallons de liqueur dans le tems ſec de Mars ou d'Avril, les cylindres qui preſſent les cannes doivent tourner deux fois et demi dans une minute, comme pour en donner deux cent dans le mois de Janvier.

Il faut remarquer auſſi que la difference du produit des deux moulins dont je viens de parler, ſuppoſant neceſſairement une différence egale ſoit dans les reſiſtances vaincues, ſoit dans les tems employés à les vaincre; la reſiſtance qu'il y auroit à vaincre dans un nouveau moulin qu'on auroit deſſein de rendre equivalent à un excellent moulin à l'eau, devroit donc être ſuppoſée d'environ 38 milliers, ou bien la puiſſance deſtinée à la vaincre, devroit

par-

parcourir dans une heure, l'espace que les mulets du moulin ordinaire à bestiaux, parcourent en deux.

Si le peu de principes que je viens d'établir, eussent été connus, il est probable que les auteurs de beaucoup de moulins proposés n'eussent pas perdu leur tems à les imaginer, ou que differentes Academies en eussent moins perdu à en faire l'examen.

XIX. *Account of an Oſſification of the Thoracic Duct* By Richard Browne Cheſton, *Surgeon to the Infirmary at* Glouceſter; *communicated by Mr.* Henry Watſon, *Surgeon to the* Weſtminſter Hoſpital.

Read April 20, 1780.

JAMES JONES, twenty-two years of age, was admitted into the Glouceſter Infirmary, June the 5th, 1779, for very troubleſome pains in his back and hip, which, from every circumſtance of his deſcription, were ſuppoſed rheumatic.

Upon a particular examination a few days afterwards, the right hip was obſerved to be fuller than it ſhould be; but the thigh ſeemed very little altered from its natural ſtate. A bliſter was applied over the joint, and the uſual anti-rheumatic remedies were preſcribed by the phyſician under whoſe care he was admitted. In about a fortnight he complained of a violent pain in his knee, for which another bliſter was ordered to the head of the fibula. During this time he could move about the ward

by the aſſiſtance of a ſtick, but ſoon after not without crutches.

His thigh now increaſed in bulk, and became œdematous; for which reaſon another bliſter was applied about the middle of it, and his knee getting into a contracted ſtate, a volatile liniment was rubbed on that part. As he now could not move about, even with the aſſiſtance of crutches, he took to his bed altogether.

From this time the enlargement of his thigh advanced very faſt, and his knee became contracted in proportion; ſo that the thigh had got into the ſame kind of relaxed poſition the limb is frequently placed in when the bone is fractured.

Soon afterwards he began to find ſome difficulty in the diſcharge of his urine, which by degrees increaſed ſo much, that the medicines preſcribed for his relief in this particular, not having the deſired effect, and his belly appearing to be diſtended from this cauſe, a catheter was attempted to be introduced, but it could not be made to paſs the neck of the bladder: a bougie, however, entered the bladder with eaſe, and ſome water came off upon withdrawing it.

From this period, by means of very great exertions of the abdominal muſcles, and by occaſional preſſure externally, he uſed to diſcharge his urine; but it came away

in ſmall quantities only at a time, and ſeemed to empty the bladder but very little; for a tumor, which for many days had been perceptible on the left ſide, and which evidently contained a fluid, afforded the uſual feel of a diſtended bladder.

He now ſuffered very violent pains all over the abdomen, but particularly about the region of the pubis, ſo that he could not bear the preſſure uſually made for ſome time paſt to force off his urine; and, as the catheter could not enter the bladder, and the introduction of the bougie was of very little ſervice, his urine now began to dribble away involuntarily.

His fever, at intervals, was very conſiderable, his ſtrength failed him very faſt, and he received no benefit from any medicines but opiates. In this moſt deplorable condition he languiſhed till the beginning of October, when the violence of his pains began to remit, and he gradually drooped into a ſtate of inſenſibility till the tenth of October, when he died.

DISSECTION.

The integuments of the abdomen felt harſh and dry, like a piece of parchment. The veins were much enlarged, and their branches could be traced all over its

ſurface in a very diſtinct manner. On the left ſide there ſtill remained an evident fullneſs, which puſhed the integuments forwards to the ſize of two fiſts, and which contained a fluid in conſiderable quantity.

The thigh ſtill continued in the ſame poſition and œdematous ſtate before deſcribed, ſo that from thoſe circumſtances, and the diſtenſion of the abdomen, nothing particular could be obſerved externally, at the lower part of the abdomen, on the right ſide more than on the left.

Upon examining the cavity the inteſtines preſented themſelves nearly in their natural ſituation: their appearance was ſound, but in general they were much inflated. The tumor on the left ſide proved to be the bladder diſtended with urine, ſlightly adhering to the peritoneum; and this, together with the colon paſſing on to its termination in the rectum, filled up the iliac region on that ſide, while the right ſide, and indeed more than half the pelvis, was fully occupied by a confuſed irregular maſs, ſeemingly formed of ſcirrhous cartilage, bone, and ſtone.

As a large cartilaginous ſubſtance [a] ariſing from this maſs ſeemed to cover the bodies of the vertebræ, I removed the inteſtines, and purſuing this ſingular appear-

(a) This ſubſtance, when firſt taken out of the body, appeared cartilaginous; but when dried was perfect bone.

ance,

ance, traced it upwards in the courſe of the ſpine, and of the large blood veſſels, to its termination ſomewhat above the kidneys.

Upon laying bare the ſternum and ribs, to inſpect the cavity of the thorax, the cartilages preſented themſelves in the whiteſt ſtate I ever ſaw, approaching nearly to the colour of writing paper, ſtill retaining their natural firmneſs and texture.

The lungs were in a full ſtate of diſtenſion, and ſtudded in many parts with the ſame kind of cartilaginous ſubſtance which appeared ſo plentifully in the lower cavity.

Suſpecting that the arterial ſyſtem might, in ſome meaſure, be affected by this prevailing diſeaſe, I ſeparated the heart from the lungs, for the purpoſe of examining its larger ſyſtem of veſſels, and, dividing the aorta juſt below its curvature, found, upon examination, the heart very flaccid, empty, and of a much ſmaller ſize than uſual in adults of the age of this patient, but ſound in every reſpect.

The ſemi-lunar and mitral valves were not at all diſeaſed, nor was there the ſmalleſt deviation in any part of the aorta from the moſt healthy ſtate, though it was intirely ſurrounded by this ſingular ſubſtance from the

paſſing

paſſing off of the cœliac artery to where it bifurcates into the two iliacs.

On raiſing the aorta from the ſpine, after I had divided it at its curvature, I found a ſingular firmneſs on its right ſide, like a piece of hard pack-thread, and exactly in the ſituation of the thoracic duct. I continued my diſſection therefore with great caution, and at ſuch an extent as to take in the vena azygos, and afterwards found upon clearing it at my leiſure, that it really was the thoracic duct, intirely plugged up with oſſific matter, from immediately above the receptaculum chyli.

Unfortunately, being much ſtreightened for time when I opened the body, I was obliged to take out the parts for a more careful examination at home, which deprived me of the opportunity of aſcertaining the above circumſtances at the time I ſeparated the parts, and conſequently of inquiring how much further up this ſingular oſſification extended, and in what ſtate the duct might be at its entrance into the ſub-clavian vein. The vena azygos, as well as the aorta, was perfectly ſound.

The vena cava was not ſo free from diſeaſe; for though it bore externally a natural appearance, when I laid it open from finding a ſingular feel within, I found that its cavity was above half filled with a firm melaſtic ſubſtance:

this

this ſubſtance originated from its internal ſurface near the entrance of the emulgent vein, attached to it here and there by ſmall points, till about the projection of the ſacrum, where the cavity of the vein was almoſt filled up with a continuation of the ſame ſubſtance.

The ſpleen, pancreas, and liver, were perfectly ſound: the gall bladder very ſmall, and quite empty; its ducts in a natural ſtate.

The kidnies were much increaſed in their ſubſtance, externally more livid than uſual, and ſeemingly in a ſtate of inflammation. The ureter on the right ſide was much enlarged, and contained a conſiderable quantity of urine, which ſeemed retained there by the diſtenſion of the bladder. The left ureter was of its natural ſize and appearance. The coats of the bladder were conſiderably thickened, but preſerving externally as well as internally its moſt healthy appearance.

It may be neceſſary here to remark, that as the bladder could not expand itſelf laterally, it was extended upwards in an oblong form, not unlike that of a calf's, but did not appear capable of containing more than a quart.

The tumor which occupied the right iliac region, extended itſelf irregularly in all directions, and appeared outwardly to have deſtroyed or brought on an abſorption of

of a principal part of the os innominatum, ſo far as this could be aſcertained by thruſting a probe into it in different directions.

The manner in which a portion of this tumor preſſed on the neck of the bladder readily accounts for the difficulty of paſſing the catheter, though a bougie eaſily ſlipped in, and gives us a reaſon why the patient was unable to empty his bladder for ſo long a time before his death.

The degree of injury the os innominatum has ſuſtained cannot yet be certified, the tumor not being ſufficiently reduced by maceration. Where the bone is cleared from the ſurrounding ſoft parts, it appears to have ſuffered a great loſs of ſubſtance, and, as the tumor diſſolves, a large quantity of bony matter now deprived of its connecting medium is continually ſubſiding to the bottom of the veſſel in which the tumor is macerating.

The preparation of the thoracic duct was at firſt put up in ſpirits to preſerve its original appearance, and in this ſtate I brought it with me to London. Upon ſhewing it here to ſeveral anatomical gentlemen, they were in doubt whether there might not be yet remaining ſome ſmall cavity in the duct, and were therefore deſirous I ſhould take the preparation out of the ſpirits to be examined more accurately. This I readily complied with,

and

and it was accordingly examined very circumſtantially by Dr. HUNTER, Mr. WATSON, Mr. J. HUNTER, and Mr. CRUIKSHANKS, who honoured me with a viſit for that purpoſe.

The appearances which theſe gentlemen particularly noticed were, that the duct was completely filled up, excepting at the lower bulbous part, commonly called the receptaculum chyli, where, indeed, there was room enough for air to paſs between the coat of the duct and the adventitious ſubſtance within it; ſo that the receptacle, which before appeared flat, upon throwing in air became rounded and fully diſtended: but this air was totally confined to the receptacle, and could not be forced up the duct in the ſmalleſt degree. The receptacle was then ſlit open, and an attempt made to paſs a briſtle up the duct, but this was found impoſſible.

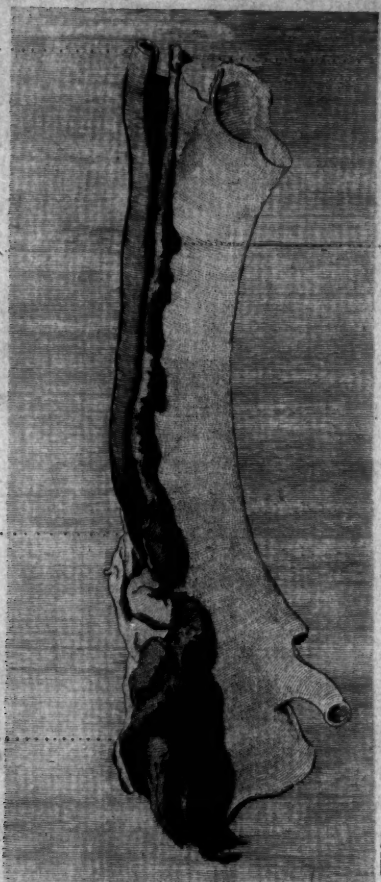
Mr. CRUIKSHANKS afterwards endeavoured to force mercury up the duct, but not the ſmalleſt particle would paſs.

From theſe different examinations we were all thoroughly convinced, that the receptaculum chyli was not ſo completely filled up but that it might receive a ſmall quantity of fluid, yet the duct itſelf was totally impervious, without a poſſibility of admitting any.

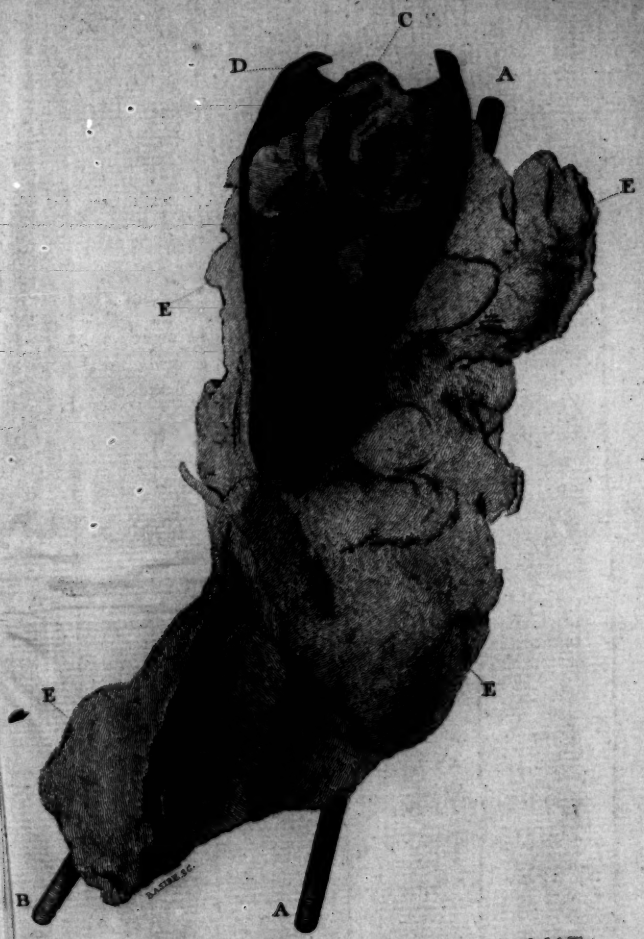
The coats of the duct did not appear to have undergone any morbid change: for in ſome places where the ſubſtance it contained was not ſo ſtrongly attached but that the coat would admit of being raiſed from it, they were found in a perfect natural condition. At other places, where the attachment was inſeparable, there was a greater appearance of offification externally; but this we were convinced aroſe merely from the thinneſs of the coats in that part, where the receptaculum chyli was laid open, the ſubſtance within it appeared of a membraneous nature very ſimilar to that found in the vena cava of this ſame ſubject, but more laminated. We preſumed, that the ſame kind of membrane had been continued through the whole of the duct, but was now become pretty completely offified in all that portion of the duct which we were in poſſeſſion of.

A ſmall body, reſembling a lymphatic gland on the ſide of the upper part of the duct, was opened by Mr. J. HUNTER, who found the ſame offific diſpoſition in this little body, as we before noticed in the duct itſelf.

We next examined the ſubſtance that partly filled up the vena cava. It was in length about four inches; at the upper end, broad and conical; at the lower, much narrower and rather rounded. Its ſurface appeared irregular and granulated with ſmall bony particles. It



A. *The Receptaculum Chyli laid open to shew the ligamentous Substance within it.*
B. *An Opening into the Thoracic Duct to shew the manner in which it was block'd up.*
C. *The little lymphatic Gland containing a similar Substance to that in the Receptac. Chyli.*
D. *The Coats of the Duct separated from the bony Matter with which it was compleatly fill'd.*
E. *Vena Azygos.*



A. A Bougie in the descending Aorta, the lower point having passed into the left Iliac.
B. A red Bougie in the lower part of the Vena Cava.
C. The Ligamentous Substance filling up the Vena Cava. D.
E. The bony Matter surrounding the Aorta & Vena Cava.

appeared flattened, perhaps from the preſſure of the blood conſtantly moving over it; for having made a ſmall opening with a lancet into the narrow part of this ſubſtance, we could paſs a probe very readily both upwards and downwards, ſo as to convince us, it was really hollow all the way. Upon introducing a blow-pipe, the upper part was expanded into a large *cul de ſac*, and the lower diſtended pretty much like a large veſſel. The appearance of this man during his illneſs, as well as at the time of his death, was exactly ſimilar to what I had frequently obſerved in patients who had lingered under, and been deſtroyed by, ſlow inflammations of the viſcera. His complaints in the abdomen only indicated a diſeaſed bladder, and, for that reaſon, I opened him: for he was not even ſo much emaciated as we often find patients under that diſeaſe.

XX. *An Account of the Effect of Electricity in ſhortening Wires.* *By Mr.* Edward Nairne, *F. R. S.*

Read May 11, 1780.

EVERY new fact in a ſcience being an acquiſition to it, I beg leave to lay before this Society the following diſcovery, which I made ſome time ago, relative to electricity ſhortening wire by its paſſage through it. The following experiment I hope will clearly illuſtrate it.

EXPERIMENT.

I took a piece of hard-drawn iron wire, ten inches long, and one hundredth of an inch in diameter. This wire was held in a perpendicular poſition between two braſs pinchers, the upper one of which was connected with a glaſs pillar, in order that the whole charge of an electrical battery might paſs only through the wire faſtened between them. Theſe upper pinchers were moveable, for the ſake of ſlackening the wire occaſionally; and, in the experiment, they were fixed with a ſcrew,

a ſcrew, ſo that the wire hung ſomewhat looſe between them and the lower ones. I then charged a battery, containing twenty-ſix feet of coated ſurface, till the index of the electrometer was raiſed to fifty degrees: it was then diſcharged through the wire, and, immediately after, the wire was ſeen to ſhorten by its drawing nearer to a ſtraight line between the fixed pinchers. If the wire was put ſo looſe that the moving of the upper pinchers three quarters of a tenth of an inch, or ſeventy-five thouſandths, would draw it juſt ſtraight, one diſcharge of the battery through it would then draw it to a ſtraight line. I diſcharged the ſame battery nine times through a piece of the ſame wire, which was alſo of the ſame length, which was ſlackened each time before the diſcharge went through it. There were preſent Sir JOHN PRINGLE, the hon. Mr. CAVENDISH, Mr. SMEATON, the Rev. Mr. MICHEL, Mr. RUSSEL, and Mr. WHITEHURST. After the ſixth and ninth time Mr. SMEATON meaſured the wire, and found it to have ſhortened in the proportion of three quarters of a tenth of an inch each time.

I afterwards diſcharged the battery ſix times more through the ſame piece of wire, which made fifteen times in the whole, and found it had continued ſhortening nearly in the ſame proportion, the wire having been ſhortened by the fifteen ſtrokes full one inch and one tenth,

tenth, *viz.* its whole length being now barely eight inches and nine tenths inſtead of ten inches that it was at firſt. I then weighed it in a pair of ſcales that would turn with leſs than the eighth part of a grain; but could not perceive, that there was any ſenſible difference in the weight. I tried it with a pair of callipers, and it ſeemed to be rather thicker.

I intended to have paſſed ſeveral more diſcharges of the battery through the wire, but the ſixteenth melted it.

I have likewiſe ſhewn this extraordinary effect of electricity to Dr. PRIESTLEY, Mr. MAGELLAN, and ſeveral other gentlemen, in whoſe preſence I paſſed ſix diſcharges of the ſame battery through a piece of the ſame wire, exactly ten inches in length; and on meaſuring it, it was found to be ſhortened four tenths and a half of an inch, which is juſt three quarters of a tenth for each diſcharge. Dr. PRIESTLEY then took a piece of the ſame wire, of exactly the ſame length, and heated it red-hot in the common culinary fire. This wire being afterwards meaſured was found to continue of its original length of ten inches, not being in the leaſt ſhortened.

I have generally found, that if the iron wire was firſt annealed in the culinary fire, the ſame ſtrength of charge melted it.

I have

I have alſo tried a piece as copper wire, gilt with ſilver, and of the ſame dimenſions as the iron wire, and find, that the ſame charge, diſcharged through it, ſhortens it one twentieth of an inch, *viz.* two-thirds of what the iron wire was ſhortened. The ſame ſtrength of electrical charge, as near as could be meaſured by an electrometer, which paſſing through the iron wire, made it viſibly red-hot in a bright day, the ſun ſhining at the ſame time, did not heat a piece of copper wire of the ſame dimenſions ſo as to appear of a red heat, though the room was made dark. If the battery was but a little more charged, the iron wire then would be melted; but it had no ſuch effect on the copper wire.

This ſeems to point out, that iron wire reſiſts the paſſage of the electric fluid much more than copper; and alſo, that the culinary fire and electrical fire have different effects on iron and copper: for malleable iron, I am informed, is one of the moſt difficult metals to melt by the culinary fire, and requires a much greater heat to melt than copper; whereas, on the contrary, the iron is melted with a much leſs charge of electric fire.

XXI. *Aſtronomical Obſervations on the periodical Star in Collo Ceti* [a]. *By Mr.* William Herſchel, *of* Bath; *communicated by Dr.* Watſon, *Jun. of* Bath, *F. R. S.*

Read May 11, 1780.

THIS remarkable ſtar, we are told [b], "was firſt ob-"ſerved by DAVID FABRICIUS, the 13th of Au-"guſt, 1596, who called it the ſtella mira, or wonder-"ful ſtar; which has been ſince found to appear and "diſappear, periodically, ſeven times in ſix years, conti-"nuing in the greateſt luſtre for fifteen days together, "and is never quite extinguiſhed."

My own obſervations on this wonderful ſtar are but few, yet ſufficiently verify the ſurprizing appearances that have been aſcribed to it. I ſhall tranſcribe them from my aſtronomical journal in the order they were made.

October 20, 1777, I looked out for the periodical ſtar in collo ceti, but it was not viſible. If its period is 312

(a) BAYER's character for this ſtar is *o*.
(b) See FERGUSON's Aſtronomy. § 366.

days I may expect to see it about Christmas, not being visible at present.

Dec. 18, 1777, I saw the periodical star in collo ceti. It appeared in the very place where, about a fortnight ago, I imagined (but was not sure) there was a faint appearance of it. It was in magnitude about equal to ζ, but not so large as δ[c].

Jan. 26, 1778, The periodical star was larger than δ, but less than γ. Being taken up with other observations, I paid no more attention to it during the rest of this period.

Sept. 18, 1779, The periodical star was visible to the naked eye, when I first looked for it.

Oct. 6, 1779, The periodical star was exceedingly bright this evening. It exceeded α and β ceti; which latter, I must here observe, is considerably larger than the former, and affords a proof of the change in the magnitude of the fixed stars; as we can hardly suppose BAYER should have made a mistake in the magnitude of the two first stars of this constellation.

The apparent magnitude of o ceti was not round but elliptical, when observed through the telescope, and not very well defined; but as it was too near the horizon,

(c) ζ is marked by BAYER as a star of the fourth magnitude, and δ of the third.

this ſhape might ariſe from that cauſe, for other low ſtars were alſo irregular in their forms, yet bellatrix was exceedingly fine and quite round.

Oct. 7, 1779, The periodical ſtar was perfectly round in the teleſcopes, and its apparent diameter well defined, full, and very large, for a ſtar of that magnitude.

When I ſpeak of the apparent diameter, I would be underſtood to diſtinguiſh it not only from the real diameter, but alſo (if I may be allowed the expreſſion) from the real apparent diameter. To explain this a little more at large: the body of the ſun, for inſtance, is of a certain dimenſion which we call his real diameter, and this remains always the ſame. His apparent diameter (which I here call real apparent) is changeable, according as we approach to, or recede from, him, and is between 31′ 33″. and 32′ 39″; but were he removed to the diſtance of one of the neareſt fixed ſtars, neither his real, nor real apparent diameter, could then be known to us by any method we have hitherto been acquainted with: for at the diſtance of at leaſt 20 billions of miles, his real apparent diameter could not much exceed thirty fourths of a degree; and a teleſcope muſt magnify above fourteen thouſand times to make him appear of only two minutes in diameter, which ſtill is hardly ſufficiently large to diſtinguiſh a ſquare from a circle: and yet I doubt not, but

that we ſhould obſerve ſome apparent diameter or other of the ſun thus removed from us; and this is what I here have called the apparent diameter. This muſt be owing to ſome optical deception. DE LA LANDE explains it thus: "Si l'on voit dans les lunettes une lumière éparſe "qui environne les étoiles, qui les amplifie et les fait "paroitre come ſi elles avoient 5 a 6″ de diametre, on "doit attribuer cette apparence à la vivacité de leur lu-"mière, à l'air environnant et illuminé, à l'aberration des "verres, à l'impreſſion trop vive qui ſe fait ſur la rétine."

Oct. 19, 1779, The periodical ſtar preceded a very obſcure teleſcopic ſtar at the diſtance of 1′ 45″.16. This meaſure was extremely difficult to take, the ſmall ſtar being ſo obſcure as hardly to bear the field of view to be ſufficiently enlightened: both alſo having nearly the ſame declination, they therefore paſs the parallel hairs of the micrometer almoſt at rectangles. However, I ſhould ſuppoſe, the meaſure muſt be true to 3 or 4″.

I meaſured the diſtance once more, with a better light and more ſatisfaction to myſelf, and found it 1′ 50″.47.

12 o' clock, The periodical ſtar is now about the meridian, and brighter than α arietis.

Oct. 30, 1779, 9 o'clock, The periodical ſtar is ſtill increaſed, and viſibly larger than α arietis, though not in the meridian at preſent.

— o'clock, *o* ceti being now in the meridian, is almoſt of a middle ſize between aldebaran and α arietis. Its apparent diameter by the teleſcope is alſo increaſed.

Nov. 2, 1779, The luſtre of the periodical ſtar is ſtill increaſed. The body is very full and round in the teleſcope. I magnified it 449 times very diſtinctly, the evening being ſo fine; but my uſual power is only 222.

Nov. 20, 1779, The periodical ſtar ſeemed to be as bright as before, but no brighter.

Nov. 30, 1779, *o* ceti is conſiderably decreaſed. Its magnitude is leſs than β but greater than α. I have before remarked, that α is leſs than β.

Dec. 4, 1779, The luſtre of *o* ceti is only equal to α. I meaſured the diſtance of this ſtar from the obſcure one which follows it:

Firſt meaſure,	1′ 53″.437
Second meaſure,	1′ 50″.625

The weather was ſo cold, that I could hardly finiſh this laſt meaſure; but I believe it to be too ſmall. The diſagreement is owing to the difficulty, and not to want of accuracy in the micrometer.

Jan. 4, 1780, The periodical ſtar is very much diminiſhed. I took the diſtance, but the evening was very indifferent, and therefore the ſmall ſtar ſo faint as to allow but little light. It meaſured 1′ 45″.937. I thought the

the meaſure too little when I took it, but could not ſucceed better.

Feb. 7, 1780, The periodical ſtar was inviſible to the naked eye. I was but little prepared to look a long time for it with the teleſcope; but ſuppoſe I ſhall be able to find it another time.

MAUPERTUIS accounts for the periodical appearances of changeable ſtars, by ſuppoſing that they may be of a flat form, like Saturn's ring, which becomes inviſible when the edge is preſented to us.

As the periodical ſtar in collo ceti appeared always full and round when I viewed it with a teleſcope, this might at firſt appear to contradict the ſuppoſition of MAUPERTUIS; but, upon proper conſideration, will be found not to be at all againſt it: for, ſuppoſe the real apparent diameter of this ſtar to be one-third of a degree; then, ſince it appeared to me (I did not meaſure it) at leaſt of one ſecond, when at the full, it will follow, that there was an aberration whatever might be the cauſe of it, which amounted to 59″, by which its real apparent diameter was increaſed from 1‴ to 1″. Now, if this ſtar, in one certain poſition, ſhould preſent its circular diſk of 1‴ in diameter, and in another ſituation only its flat edge which would appear as a line of 1‴ in length, both appearances, with the aberration included will ſtill remain

remain of a circular form: for, adding the aberration 59‴ to the length 1‴, the whole becomes 1″; and adding 59‴ to ſcarce any breadth at all, the whole breadth will alſo become nearly 1″.

KEILL ſays, " It is probable, that the greateſt part of " this ſtar is covered with ſpots and dark bodies, ſome " part thereof remaining lucid; and while it turns about " its axis, does ſometimes ſhew its bright part, ſome- " times it turns its dark ſide to us; but the very ſpots " themſelves of this ſtar are liable to changes, for it does " not every year appear with the ſame luſtre. Some- " times it reſembles a ſtar of the ſecond magnitude; in " other years in can ſcarcely be reckoned among ſtars of " the third order; nor are the times of its viſiting us " always of the ſame duration."

XXII. *An Account of a new and cheap Method of preparing Pot-aſh, with Obſervations.* *By* Thomas Percival, *M. D. F. R. and A. S. Member of the Royal Society of Phyſicians at* Paris, *&c.*

Read April 6, 1780.

THE agriculture ſociety at Mancheſter has long recommended the making of reſervoirs for the water which flows from dung-hills in farm yards. This water is ſtrongly impregnated with the ſalts and putrid matter of the dung-hill, and by ſtagnation it acquires a much higher degree of putreſcence, and probably becomes proportionably more replete with ſalts. When thus collected and improved, it is pumped into an hogſhead, which being drawn upon a ſledge, or ſmall cart, is conveyed into the meadows, for the purpoſe of ſprinkling them with this rich manure. This important improvement in rural oeconomy, I apprehend, has not been extended much beyond the diſtrict of our ſociety; and it ſeems to be unknown to one of the lateſt and moſt intelligent writers on huſbandry: for Lord KAIMS, in a recent

cent work on this ſubject, of which he has favoured me with a copy, has not even mentioned it.

But theſe reſervoirs may be applied to a purpoſe ſtill more ſubſervient to public utility than that above deſcribed. JOSIAH BIRCH, Eſq. a gentleman who carries on an extenſive manufactory, and bleaches his own yarn, about ſix months ago, was induced, by a happy turn of thought, to try whether the dung-hill water might not be converted into pot-aſhes. He accordingly evaporated a large quantity of it, and burnt the *reſiduum* in an oven; the product of which ſo perfectly anſwered his expectations, that he has ever ſince continued to prepare theſe aſhes, and to employ them in the operations of bucking. A ſtranger to that narrowneſs of ſpirit which ſeeks the concealment of a lucrative diſcovery, he is deſirous that it ſhould be communicated to the Royal Society, and has furniſhed me with the following account, together with the plan annexed.

"The quantity of muck water uſed was twenty-four "wine pipes full, which employed a man and two "horſes two days, to cart it from the pump to the pan "wherein it was boiled; but this expence I ſhall now "ſave, as I ſhall lay a ſough of brick, which will convey "it from the pump to the boiler. The coals uſed to "boil and burn it were one hundred and twenty baſkets,

" and I ſuppoſe each baſket weighs ſix ſcore pounds or " upwards. One man was occupied three weeks in boil- " ing and burning. The quantity of aſhes made was " 9 cwt. 1 qr. 12 lbs. well worth, at the preſent price of " aſhes here, two guineas *per* hundred.

	L.	s.	d.
" 9 cwt. 1 q. 12 lb. at 42 s. *per* cwt.	19	13	0
" A man and two horſes, two " days, at 6 *s.* *per* day,	0	12	0
" 120 baſkets of coals, at 5 d. " *per* baſket,	2	10	0
" A man's wages for three weeks,	1	7	0
	4	9	0
	15	4	0

" The profit, therefore, amounts to 15 l. 4 s. deduct- " ing only a trifle for the wear of the pan and oven."

The profits ariſing from this preparation of pot-aſh are ſufficiently evinced by the foregoing eſtimate; and they may, perhaps, admit of increaſe by future improvements. In the ſpring and ſummer ſeaſons, I ſhould apprehend, the evaporation might be carried on without the aid of fire, by conveying the dung-hill water from the reſervoir through proper ſluices into ſhallow troughs or ponds, of ſuch extent as to afford a ſufficient

ſurface for the action of the ſun and wind[a]. Theſe might be covered in rainy weather with awnings of canvas, painted on the outſide black, and white on the inſide; the former with a view to abſorb, the latter to reflect, the rays of light.

(a) The following abridged view of a meteorological regiſter, which I kept with great exactneſs during the years 1774 and 1775, may throw ſome light on the practicability of this plan in the climate of Lancaſhire, which, I believe, is nearly the ſame as that of moſt of the other Weſtern counties of England.

1774	Thermometer		Days	
	2 o'clock.	P.M.	Rainy	Fair.
	°	°		
Jan. Feb. March,	56	28	25	65
April, May, June,	72	45	55	36
July, Auguſt, Sept.	75	53	66	26
Oct. Nov. Dec.	60	30	43	49
1775				
Jan. Feb. March,	54	30	61	29
April, May, June,	78	51	42	49
July, Auguſt, Sept.	74	48	62	30
Oct. Nov. Dec.	64	32	50*	28*

* Fourteen days omitted, no account being taken.

The thermometer, uſed in making the preceding obſervations, was graduated according to the ſcale of FAHRENHEIT. It was placed in the open air, and in a Northern expoſure. The column of rainy days expreſſes the leaſt as well as the greateſt quantity of rain. The column of fair days includes only thoſe days in which not a ſingle ſhower was noticed. The day comprehends twenty-four hours.

The mean quantity of rain which annually falls here is about 39 inches.

This

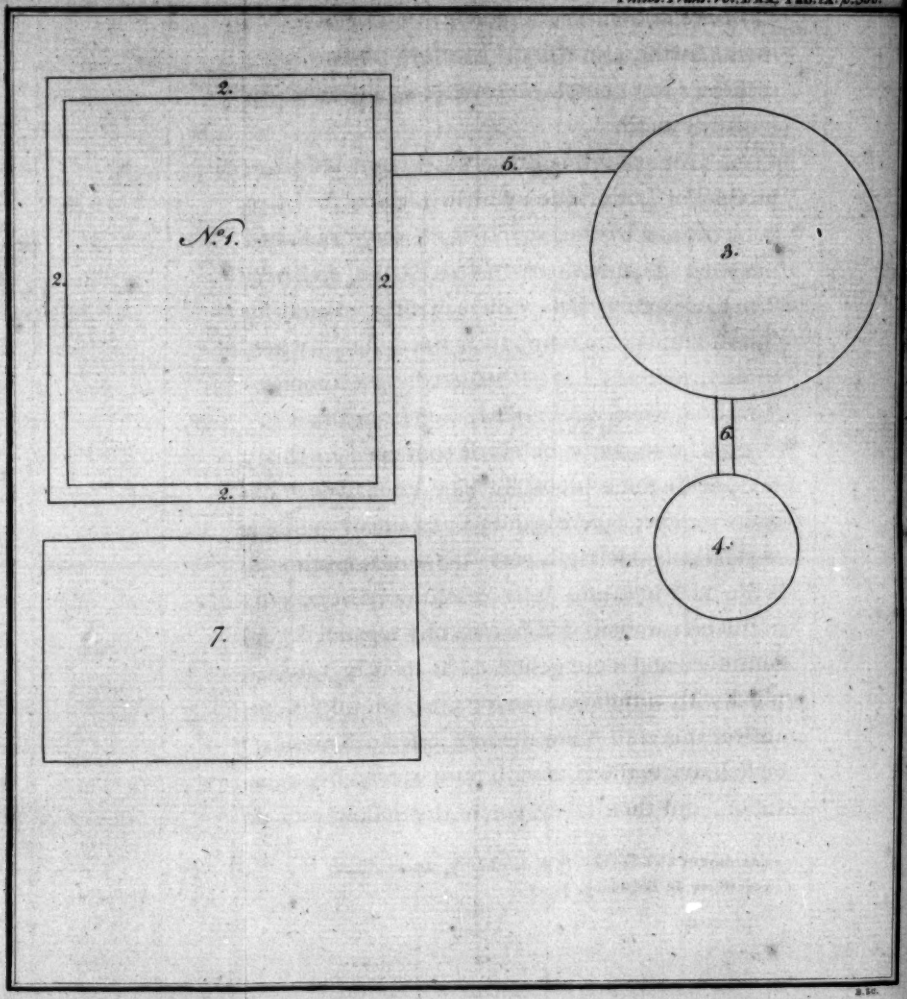
This pot-aſh is of a greyiſh white appearance, deliqueſces a little in moiſt air; but if kept in a dry room, near the fire, acquires a powdery ſurface. It is hard, and of a ſpongy texture when broken, with many ſmall cryſtals in its ſubſtance. The colour of its internal parts is duſky and variegated. To the taſte it is acrid, ſaline, and ſulphureous. It emits no ſmell of volatile alkali, either in a ſolid form, diſſolved, or when added to lime-water; neither does it communicate the ſapphire colour to a ſolution of blue vitriol. Silver is quickly tinged black by it, a proof that it contains much phlogiſton. Ten grains of this pot-aſh required eleven drops of the weak ſpirit of vitriol to ſaturate them. The like quantity of ſalt of tartar required, of the ſame acid, twenty-four drops: a ſtrong effervеſcence occurred in both mixtures; from the former a ſulphureous vapour was exhaled. A tea ſpoonful of the ſyrup of violets, diluted with an ounce of water, was changed into a bright green colour by five grains of the ſalt of tartar; but ten grains of this pot-aſh were neceſſary to produce the ſame hue, in a ſimilar mixture. Half an ounce of the pot-aſh diſſolved intirely in half a pint of hot water; but when the liquor was cold, a large purple ſediment ſubſided to the bottom: and it was found, that this ſediment amounted to about two-thirds of the whole quantity of aſhes uſed.

I have not leiſure, at preſent, to proſecute theſe experiments farther, and ſhall therefore content myſelf with making a few general obſervations on the facts that have been advanced.

1. This pot-aſh is a true fixed vegetable alkali, and a product of putrefaction which has not, that I recollect, been noticed by the chymiſts. A very celebrated writer has even in expreſs terms aſſerted, that "all vegetables, "not excepting thoſe which in their natural ſtate fur-"niſh aſhes containing much fixed alkali, when burnt, "after their acid has been altered by a complete putre-"faction, leave aſhes intirely free from alkali (b).

2. The quantity of alkali contained in this pot-aſh may, with ſome probability, be eſtimated at one-third of its weight; whereas the white Muſcovy aſhes are ſaid to yield only one-eigth part (c). Of its impurities ſulphur is the moſt injurious to its bleaching powers, and ſhould in the preparation of it be carefully ſeparated. A longer continued and more gentle calcination in a furnace ſupplied with a ſufficient current of air might, perhaps, anſwer this end. But the moſt effectual method would be to lixiviate the ſalts with pure water, after a moderate fuſion, and then to evaporate them ſlowly to dryneſs.

(b) MACQUER's Dictionary of Chemiſtry, article Alkali.
(c) HOME on Bleaching, p. 157.


2.
2.
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It muſt, however, be remarked, that in thus freeing the pot-aſh from phlogiſtic matter, another impurity is generated: for both the action of fire, and the ſolution in water, convert into earth a portion of the alkaline ſalt.

3. No quick-lime appears to be contained in this pot-aſh: for a ſolution of it, poured from its ſediment, remained clear, though long expoſed to the air. Nor did it acquire any milkineſs by being blown into from the lungs; but perhaps the addition of this cauſtic ſubſtance would increaſe its activity and value, when employed in ſoap-boiling and other arts: for the Ruſſian pot-aſh is more pungent to the taſte, ſaturates a larger proportion of acid, and diſſolves oils more powerfully than the pure alkaline ſalts. And Dr. HOME has proved (d), that theſe qualities depend on a large admixture of quick-lime.

4. It would be worthy of trial, to aſcertain whether the large purple ſediment, which ſubſides when this pot-aſh is lixiviated, might not be applied to the manufacture of Pruſſian blue; or uſed in the manner recommended by Mr. MACQUER for dying wool and ſilk. See the Memoirs of the French Academy for the year 1749 (e).

5. The farmer, though he live at a diſtance from the manufactures in which pot-aſh is employed, may yet

(d) Eſſay on Bleaching. NEUMANN's Chemiſtry, by LEWIS.
(e) See alſo NEUMAN's Chemiſtry, by LEWIS, p. 73.

find

find his account in preparing it from dung-hill water: for it will furnish him with a top dressing for his garden and land, of great fertilizing powers. But if fuel be dear where he resides, and conveniences for combustion be wanting, the simple evaporation of the water may suffice, and the putrid lye, thus reduced to a solid form, will prove to be a rich manure. At Hart Hill, my summer abode, about three miles from Manchester, I have lately practised a method of making a compost of the dung-hill water. The weeds and rakings of the garden, the dressings of the fields, the leaves blown from the trees, and other refuse matters, are put together near the reservoir, out of which the water is occasionally pumped and scattered over the heap: so strong a ferment almost instantly excites putrefaction. And these vegetable substances are soon converted into a fertile mould, which retaining the salts and oils of the dung-hill water, suffers the superfluous moisture to exhale into the air, or to percolate through it. And I have found, by experience, that the compost thus prepared is laid on the meadows at less expence, and that it is more efficacious and durable in its operation than the sprinklings, which, at stated times, they formerly received: for my land, though good and in fine condition, is light and

ſandy, and the dung-hill water quickly paſſed below the roots of the vegetables which grow upon its ſurface.

Mancheſter, March 3, 1780.

EXPLANATION OF THE FIGURE.

1. The dung-hill.

2. A ſough or drain round the dung-hill.

3. A reſervoir for the dung-hill water.

4. A well, communicating with the reſervoir, in which a pump is fixed, to convey it to N° 7.

7. A pan in which the water is boiled to the conſiſtence of treacle, and afterwards burnt in an oven. The pan is made of iron plates, turned up a little at the edges. To theſe are ſcrewed planks of wood, to make it about twenty inches deep.

XXIII. *On the Degree of Salubrity of the common Air at Sea, compared with that of the Sea-ſhore, and that of Places far removed from the Sea. In a Letter from* John Ingen Houſz, *M. D. F. R. S. to Sir* John Pringle, *Bart. F. R. S. Dated* Paris, Jan. 22, 1780.

Read April 24, 1780.

SIR,

AS you had recommended to me the examination of the air at ſea by the nitrous teſt, I followed your advice in my return to the Continent in the beginning of November laſt: and I embraced that opportunity with the more eagerneſs, as I knew that you had given credit to the account of ſeveral conſumptive people having recovered their health by going on ſea voyages, after the common means for curing that diſtemper had failed.

I was in hopes likewiſe to find in this inquiry, a confirmation of what you conjectured in your Anniverſary Diſcourſe in the year 1773, *viz.* that great bodies of water, ſuch as ſeas and lakes, are conducive to the health of animals, by purifying and cleanſing the air contaminated

nated by their breathing in it: ſo that the ſalutary gales, by which this infected air is conveyed to the waters, and by them returned again to the land, though they do riſe now and then to ſtorms and hurricanes, muſt nevertheleſs induce us to trace and to revere in them the ways of a beneficent Being, who, not fortuitouſly, but with deſign, not in wrath, but in mercy, thus ſhakes the waters and the air together, to bury in the deep thoſe peſtilential effluvia which the vegetables upon the face of the earth are inſufficient to conſume.

I was not without hope, that ſuch experiments might tend to throw a new light upon the cauſe of that almoſt univerſal effect of the ſea air, to wit, its increaſing the powers of life, and giving a keener appetite by haſtening the digeſtion of food.

I ſhall now give you an account of the experiments I made in conſequence of your ſuggeſtion, in the ſame order as they were made; and beg you to preſent them to the Royal Society, if you think them worthy the attention of that learned body.

I muſt firſt, however, premiſe, that as I wrote this paper in noiſy inns, on ſhip-board, and places little adapted to philoſophical application, I hope you will make ſome allowance for the inaccuracies which you may

find in it. I began my experiments at Graveſend, where I was obliged to wait two days for a favourable wind. I found the air of that place, on Nov. 1, of a tolerable good quality, as one meaſure of it with one of nitrous air occupied, in ſeveral trials, about 104, or one meaſure and $\frac{4}{100}$ of a meaſure; ſo that I took it to be nearly of the ſame quality as the air of London.

The ſhip in which I went from London to Oſtend happened to be becalmed about two or three miles from ſhore, in the mouth of the Thames, between Sheerneſs and Margate. The weather was very agreeable, warm, and the ſun ſhone very bright, on the 3d of November. I was provided with a travelling apparatus, made on purpoſe by Mr. MARTIN, the whole of which was packed up in a box about ten inches long, five broad, and three and an half high. The glaſs tube or great meaſure, which was ſixteen inches long, and divided in two ſeparate pieces, lay in a ſmall compaſs, and could be put together by braſs ſcrews adapted to the divided extremities. Inſtead of a water trough, ſuch as is uſed commonly, I made uſe of a ſmall round wooden tub, which I found on board the ſhip, and which I filled with ſea-water, fixing to the edge of this tub, by means of a ſcrew, the braſs funnel, through which the air was to be let up into the glaſs tube.

After

After having exerciſed myſelf during ſome time in performing the experiment in a water tub much too ſmall for the purpoſe, I at laſt acquired a habit of doing it tolerably well. I then began to make my experiments regularly at about eleven o'clock; and I have the pleaſure to inform you, that I found the ſea air at the place indicated of a ſuperior purity to any common air I ever met with ſince the month of June laſt (when I began to engage in the courſe of experiments which have afforded me the materials of my work lately publiſhed upon *Vegetables*) either in my country retirement, or in London. In ſix different trials made one after another in the ſhort manner deſcribed in my book, p. 278. *et ſeq.* [a] I found, that the two meaſures of air (one of common and one of nitrous air) occupied from 0.91 to 0.94; which difference in the reſult, though but ſmall in itſelf, was owing to the diſadvantage of not having a veſſel deep enough to move the glaſs tube in with eaſe, for the purpoſe of mixing the two airs together, and to my not having yet acquired the habit of uſing the portable apparatus.

I tried alſo the air of this ſpot in the manner uſed by Abbé FONTANA, which I have deſcribed in my book,

[a] It conſiſts in letting up into the glaſs tube one meaſure of common air, and after this one meaſure of nitrous air, and ſhaking the tube forcibly in the water trough juſt at the moment the two airs come into contact with each other.

p. 155 [b]; the result of which trial was, that after the first measure of nitrous air was let up, the column of both airs occupied 1.86, or one measure and $\frac{86}{100}$ of a measure; after the second measure of nitrous air was let up, the bulk of both airs occupied 202, or two measures and $\frac{2}{100}$ of a measure: and after the third measure of nitrous air was let up, the remaining bulk of both airs occupied 2.96, or $\frac{96}{100}$ of a measure: so that the remaining bulk of both airs employed in the experiment (*viz.* two measures of common and three of nitrous air, each making 100 sub-divisions of the glass tube) amounted to two measures and $\frac{96}{100}$ of a third measure, or to 296 sub-divisions, which being subtracted from the five measures, or from the 500 sub-divisions employed, the remainder was 2.04, which was exactly the quantity of both airs destroyed.

Give me leave, before I proceed farther, to recall to your memory some of the experiments relative to this subject, made in my country retreat in the course of last summer, and related in my book, principally in p. 155. and 282. together with some further ones I made just

(b) It consists in letting up into the divided glass tube two measures of common air, and afterwards three measures of nitrous air, one after another, shaking the tube in the water trough after each measure of nitrous air, and beginning constantly this motion exactly at the moment the two airs come into contact with each other, or even before they meet, which is still better.

before my ſetting out for the continent: this recapitulation will make the nature of the experiments juſt mentioned, and their reſult, much more eaſily underſtood.

The different degrees of ſalubrity of the atmoſphere, as I found it in general in my country houſe at Southall Green, ten miles from London, from June to September, lay between 103 and 109, that is to ſay, that of the two meaſures of air, *viz.* one of common and one of nitrous air, the remaining bulk or column occupied between 103 or 109 ſub-diviſions in the glaſs tube.

I was ſomewhat ſurprized, when upon my return to town to my former lodgings in Pall Mall Court, I found the common air purer in general in October than I uſed to find it in the middle of ſummer in the country; for on the 22d of October, at nine o'clock in the morning, the weather being fair and froſty, I found, that one meaſure of common air and one of nitrous air occupied 100 ſub-diviſions in the glaſs tube, or exactly one meaſure. It gave by the Abbé FONTANA's method above mentioned the following reſult, 184, 208, 304; ſo that the quantity of both airs deſtroyed in this trial was exactly one meaſure and $\frac{96}{100}$ of a meaſure, or 196 ſub-diviſions. That very day, at two o'clock in the afternoon (it being then rainy weather) the air was ſomewhat altered for the worſe;

worſe; for at that time one meaſure of common and one of nitrous air occupied 102.

The next day, October 23, it being rainy weather, the ſtate of the atmoſphere was the ſame in the morning as it was the day before in the afternoon: for one meaſure of common and one of nitrous air occupied 102; and the reſult of Abbé FONTANA's method was as follows, 184½, 211, 307, ſo that the quantity of both airs deſtroyed amounted to 193.

October the 24th, I again examined the ſtate of the atmoſphere at nine o'clock in the morning, the weather being ſerene; I found it reſtored to its former goodneſs; for one meaſure of common and one of nitrous air occupied 100, or exactly one meaſure; and the reſult of Abbé FONTANA's method was 184, 207, 304. At ſeven o'clock in the evening of the ſame day the air was again grown worſe; for one meaſure of common and one of nitrous air occupied 103.

October 25th, I examined the air at eleven o'clock in the morning, the ſky being cloudy, and found, that one meaſure of common and one of nitrous air occupied 102. I put it again to the teſt at eleven o'clock at night, and found, from five different trials, that one meaſure of it with one of nitrous air occupied 105.

October 26th, the weather being very dark and rainy, I found that one meaſure of common and one of nitrous air occupied 105.

You know, SIR, that the ſituation of my lodgings was ſuch, that the back part of the houſe was contiguous to the garden of Carleton Houſe, which is amply furniſhed with lofty elm trees, ſo as to make the garden appear from my windows like a foreſt. As I had diſcovered, in the courſe of laſt ſummer, the great power which the leaves of trees poſſeſs of improving the atmoſphere, by pouring down an inviſible ſhower of purified or dephlogiſticated air, during the day-time, in clear weather; I could not forbear aſcribing in ſome meaſure the purity of the common air of that ſpot to the happy ſituation of the place juſt by ſo many trees, which had all kept their leaves in full vigour till that time. But I only give this as a conjecture; for I am really ſorry that I cannot prove it by a direct experiment. To put out of all controverſy ſuch a powerful influence of neighbouring trees upon the circumambient and unconfined air[c], the air of different and diſtant

(c) I ſay *unconfined*, for I am not quite ſure that the influence of trees and other vegetables, though in reality very great (as I think I have put beyond all doubt in my book) can be more ſenſible near trees in the open air than at ſome diſtance; in the ſame manner as ſome diſtilled water, poured gradually among an immenſe maſs of common water, would diffuſe itſelf ſoon through the whole maſs equally.

places

places ſhould be examined at the ſame time as the air near the trees. I was deſirous of doing it, and gave orders to my ſervant to gather air from different parts of London; but the hurry of buſineſs, occaſioned by our approaching departure, made him forget or overlook my orders; and for the ſame reaſon I myſelf forgot to perform the taſk. After all, the beſt, and perhaps only good way to clear up this matter would be, if ſeveral philoſophers, each provided with an exactly ſimilar apparatus, and living in different parts of the town, and in different parts of the country, ſhould at the ſame time put the air of the place of their abode to the teſt, and afterwards compare their notes. I conceived ſome flattering hopes before I ſet out from London, that ere long ſuch eudiometers, as I have deſcribed in my book, will come into the hands of many good philoſophers who will take upon them with pleaſure to perform this taſk, as I heard that ſome of them were already ordered to be made for many perſons, among the reſt for Dr. HEBERDEN.

Though I have ſome reaſon to believe, that the vicinity of the trees did really contribute ſomewhat to render the air purer at the above mentioned place; yet I believe alſo, that the froſty weather of itſelf contributes to purify the atmoſphere (perhaps by checking a great many

many cauſes of corruption of various ſubſtances); and that the livelineſs we commonly enjoy in froſty weather is in a great meaſure owing to the ſuperior degree of purity of our common element at that time.

I muſt now return from this digreſſion to the nature of the ſea air.

From what I have already related it appears, that the difference between the beſt atmoſpheric air I have yet found and the ſea air, as I found it by the firſt examination upon the ſpot where chance carried me, is as 91 to 100, the leſſer number indicating the beſt quality. Now, as I found the ſea air of ſuch a pure quality ſo near land, I thought it might, with ſome degree of probability, be expected, that the common air, at a diſtance from land, would prove of a ſtill ſuperior quality; for I could ſcarce believe, that in the firſt trial, made without choice of place or time, I had juſt hit upon a time and a place where the ſea air is of the firſt quality.

I would have repeated the ſame experiment next day, November the 4th, when we were in the middle of the channel between the Engliſh coaſt and Oſtend; but the motion of the ſhip, which was very great, made it impracticable. Not intirely, however, to loſe the opportunity which the voyage afforded me, I filled three phials, made on purpoſe for ſuch uſe, with

air, when we were in the middle of the ſea (the wind blowing pretty hard, and the weather being rainy). I kept theſe bottles ſhut till next day, November 5th, when I examined the air confined in them at Oſtend, and found it of an inferior quality to that which I had tried on the 3d of November, in the mouth of the Thames; for one meaſure of it with one of nitrous air occupied, in three different trials, 097. I found the common air at Oſtend near as good the ſame day at ten o'clock in the morning (the weather being cloudy); for one meaſure of it with one of nitrous air occupied 098. In the afternoon (the weather being very rainy) the common air of the place was become worſe, though ſtill of a very good quality; for one meaſure of it with one of nitrous air occupied 100, or exactly one meaſure.

About five o'clock in the evening, the weather continuing rainy, and the wind blowing at that time very hard from the ſea, I went to the ſea-ſhore on purpoſe to gather in a phial, fitted for the purpoſe, the ſea air juſt as it blew towards the land. When I had got it I went immediately home, and put it directly to the nitrous teſt. I found, by ſeveral trials of ſuch good quality, that it was nearly as good as that which I had met with in the mouth of the Thames; for one meaſure of it with one of nitrous air occupied 094 and 095,

whereas

whereas the common air of the inn (as I found by trial at the time) was ſomewhat of an inferior quality, though ſtill remarkably pure; for one meaſure of it with one of nitrous occupied 097 in five repeated trials.

As the difference in the quality of the ſea air examined on the ſpot in the mouth of the Thames, November 3d, and that which I gathered in the middle of the ſea in rainy and windy weather, was ſo remarkable, I ſuſpected the reaſon of this difference to be, that the air, put to the teſt November 3d, had been expoſed during ſeveral days to the influence of the ſea without any mixture of land air, as it had been remarkably calm all that time; and that the air gathered on the ſea in windy weather was mixed with air driven from the land by the wind, and incorporated with the ſea air. This ſuſpicion was afterwards ſtrengthened when I found the air gathered at the ſea ſhore, on the evening of November 5th, near as good as that which I gathered on a fair day in the mouth of the Thames, November 3d; for the wind being N.W. the air driven upon the coaſt was to be conſidered as true ſea air, without any mixture of land air.

But after I had made up my mind about the difference of the above related experiments, a doubt roſe in me about a circumſtance to which this difference might have been owing, at leaſt in ſome meaſure; the circumſtance I mean

was this: I had made uſe of ſea water for the experiment on the 3d of November, whereas I had made uſe of pump water in examining the ſea air kept in bottles at Oſtend. I thought I had no right to draw any concluſion from the fact till I was convinced that the making this experiment in common or in ſea water would make no difference in the reſult. This conſideration made me ſtay one day longer at Oſtend, on purpoſe to ſatisfy my mind on this head, leſt I ſhould never find another opportunity of doing it. I immediately ordered a pail full of ſea water to be brought to my lodging, and made ſeveral comparative trials with atmoſpheric air in common water and in this ſea water; but I could not obſerve any real difference in the reſult. Thus all degree of ſuſpicion about the difference of the reſult from this cauſe was now at an end.

There now only remained ſome little ſuſpicion that the air gathered on the ſea, and kept in phials, might have undergone ſome alteration; this might have been the caſe, as I had found in ſome former experiments made in England, that air kept in bottles was ſometimes liable to alterations, which I think is partly to be aſcribed to the difficulty of finding bottles ſo well ſecured by a ground ſtopper as to ſhut out every communication with the external air, and partly, perhaps, to the nature of air, which is not in itſelf an unalterable ſubſtance,

as

as I have attempted to prove in my book upon Vegetables, p. 107: in the preſent caſe, however, I rather incline to think, that the reaſon why the air, gathered in rainy and windy weather on the ſea, was found of an inferior quality, is to be aſcribed chiefly to the land air being driven by the wind upon the ſea, and thus to the mixture of both airs.

November the 6th, about nine o'clock in the morning, the weather being cold, windy, and cloudy, I again put the common air of Oſtend, which I gathered in my inn, to the teſt, and found it of near as good a quality as that in the mouth of the Thames; for one meaſure of it with one of nitrous air occupied 094½ in three repeated trials.

The ſame morning, about eleven o'clock, the wind blowing very hard from the ſea, I went to the ſhore on purpoſe to gather ſome air juſt as it came from the ſea. I found its quality inferior to that which I had examined two hours before, though ſtill ſuperior to any air I have yet found in England; for one meaſure of it with one of nitrous air occupied 097. The common air, as I found it in my lodging, was at 098. The wind had not ſhifted much, though I cannot aſcertain the exact point from which it then blew. It ſeems probable from the foregoing experiments, that though in general the ſea air

ſurpaſſes the land air in purity, yet there are the ſame inconſtancies in its degree of goodneſs as in the land air. I will not attempt to give any reaſon for this inconſtancy, as I have no experiments to ſupport any; but I think it highly deſerving the attention of philoſophers to attempt the diſcovery of this phenomenon.

This experiment being finiſhed, I cloſed my inquiries at Oſtend, and ſet off for Bruges.

We arrived at dark, and about ſeven o'clock in the evening I tried the common air of that place, and found it inferior in purity to that of Oſtend in above ten experiments: one meaſure of it with one of nitrous air occupied 105, or thereabout. I had the mortification to find the ſtoppers of the phials in which I had kept the air of Oſtend all looſened, ſo that I could not make any comparative trial with both airs.

November 8th, I ſet out for Ghent, where I ſpent the next day, November 9th. I tried the air of that place about three in the afternoon, and found it better than that of Bruges; for one meaſure of it with one of nitrous air occupied 103, or thereabout, in ſeveral experiments.

As all the following trials with common air are made in the ſame way as the foregoing, *viz.* by mixing one meaſure of common with one of nitrous air, I will hereafter only

only mention the numbers of the result, for the sake of abridging the paper by avoiding continual repetitions.

November 12. At Brussels, I found the air, at seven o'clock in the evening, at $105\frac{1}{2}$.

November 13. I found the air of the lower part of the city at 106, that of the highest part at 104; the weather was rainy and damp. It is a common opinion at Brussels, that the air in the lower parts of the city is more unwholesome than in the higher parts, and that people of bad constitutions cannot stand it unless they go to live in the higher parts.

November 14. I found the air of the lower and higher parts of the city of the same goodness, each being at 103. The weather was fair and frosty.

November 15. The air was the same as yesterday in both parts of the city; the weather fair and cold.

November 22. I arrived at Antwerp, where I found the air in the evening at $109\frac{1}{2}$; the weather was rainy, damp, and cold.

November 23. I set out for Breda. I filled a phial with air, when I set out from Antwerp at eight o'clock in the morning. I also filled a phial with air on the middle of the heath or common called *De Lange Hey:* the weather was remarkably close and damp. I tried these two airs at night, at Breda; that of Antwerp was found

at 106; that of Breda the ſame; that of the heath at $105\frac{1}{2}$.

November 24. I examined the air at Breda in the morning, about eleven o'clock, the weather being fair, cold, and inclining to froſt; it was at 102. At ſeven o'clock in the evening it was at 103.

November 25. The air at Breda was at 104; the weather rainy and cold.

November 26. The air was in the morning and in the evening 103; the weather very rainy, cold, and ſtormy.

November 27. I ſet out from Breda for Rotterdam, and croſſed the water at the Moordyke. I tried the air at the Moordyke cloſe to the water, and found it at $101\frac{1}{2}$; the weather was fair and cold, but not froſty. This ſpot is reckoned very healthy; the inhabitants have a ſound look, and live to a great age in general.

November 28. I examined the air at Rotterdam, it was at 103; the weather rainy and cold.

November 29. Being at Delpht, I gathered ſome air in the middle of the day, the weather being ſtormy and rainy. I examined it next day at the Hague, and found it at 103; and that of the Hague 104.

November 30. I examined the air at the Hague, and found it at 104; the weather cold, the wind Northerly.

December

December 1. Being ſtill at the Hague, the weather underwent a ſudden and remarkable change. The wind was ſoutherly and ſtormy; the air was become ſo warm that upon going out of the houſe into the ſtreet, I felt the ſame ſenſation as upon coming from a cold air into a room heated by a German ſtove. I ſuſpected that this ſudden change would alter the conſtitution of the atmoſphere in point of ſalubrity. Having no time to make any experiment, I contented myſelf with filling ſome phials with this air, and ſending my ſervant to Schevelingen to gather ſome air cloſe to the ſea.

December 2. The wind and weather remained the ſame as yeſterday. In the evening, about eight o'clock, when FAHRENHEIT's thermometer ſtood at 54°, I put the common air to the nitrous teſt, and found it at 116; the air gathered the day before at 117; and that gathered cloſe to the ſea at 115. As I had never found the common air near ſo bad, I had ſome apprehenſion that my eudiometer was out of order, or that ſomething was the matter with the nitrous air. I made therefore freſh nitrous air, and repeated the experiment many times, but the reſult was nearly the ſame. In the mean time, I had the following accidental meeting. The father of the landlady of the houſe having been informed by the ſervant, that I was about ſome extraordinary purſuit, of which

he could have no conception, was led to come and ſee what I did. He had ſcarce been a minute with me but I perceived he laboured under a ſevere aſthma. He explained his caſe to me, knowing me to be a phyſician, and told me, *that he had paſſed theſe two days very uncomfortably, finding the air ſo uncommonly heavy that he could ſcarce draw his breath:* which convinced me, that the element was in reality become of an inferior quality.

December 4. Being at Amſterdam, I found the air of that place at 103; the weather being rainy, windy, and cold.

December 5. The air was at 102; the weather nearly as yeſterday.

December 10. I returned to Rotterdam, and found the air at 101; the weather rainy.

In the beginning of laſt year they made an end of draining a large meer (about half the ſize of the Haerlemmer Meer) ſituated in the neighbourhood of Rotterdam, which was turfed out in former ages. It was now laid into arable land, and turned out to be very fruitful. When this land was quite cleared of the water, an uncommon epidemical diſeaſe broke out in all the places ſituated upon the borders of this lake; it began about Auguſt, and abated when the winter ſeaſon ſet in. This diſtemper broke out a-freſh laſt ſummer, and was

now again upon its decline. It carried off a great number of inhabitants. It appeared chiefly under the habit of an irregular intermittent, a bilious remitting, and a putrid fever. There was ſcarce a ſingle houſe to be found in which there were not ſome perſons ſick. The villages at a quarter of a league diſtance from the former lake were free from it. This diſtemper was aſcribed to the putrid exhalations of this newly uncovered land; which exhalations were very offenſive to the ſmell. This was ſo much the more probable, as the diſeaſe abated when the ſtench, checked by the cold, abated. I tried the air of this former lake upon the ſpot, in company with my learned friend Dr. DE MONCHY, profeſſor of phyſic, and found it as good as that of Rotterdam; but there was a great deal of wind that day, and no perceptible ſtench. However, Dr. BICKER, an eminent phyſician of that city, got me a phial filled with air of this lake, which he took from a ſpot where he ſtill perceived ſome of the former bad ſmell. This air proved to be in reality of an inferior quality to that of the city.

December 12. Being in the middle of the water between Dort and the Moordyke, I found the air upon what is called Holland's Diep of an inferior quality, the weather being remarkably dark, rainy, and windy; it was at 109.

December 13. Being returned to Breda I found the air of that place at 109 in the morning, the weather continuing as it was yesterday. In the afternoon it was somewhat better, *viz.* at $106\frac{1}{2}$, the weather having cleared up.

December 16. Having returned to Antwerp, I found the air of the lower part of that city at 105; and that of the higher part at 104, the weather being rainy and temperate.

December 17. The air at Antwerp was 107, the weather continuing to be nearly as it was the day before.

December 19. Being returned to Bruſſels, I found the air at 109, the weather being rainy, windy, and rather warm.

December 21. I found the air at Bruſſels at 106, the weather being dry and cold.

December 22. The air of Bruſſels was the ſame as yeſterday; the weather was nearly the ſame alſo.

December 23. I arrived at Mons, and found the air of that place at 104; the weather rainy and cold.

December 24. Being near Bouchain, I found the air at $104\frac{1}{2}$; the weather was cloudy and cold.

December 25. I tried the air at Peronne, and found it at $102\frac{1}{2}$; the weather was froſty.

December

December 26. Being at Cuvilli, a village four leagues from Roye, I examined the air of that place, and that which I had gathered on the road about twelve o'clock in the day-time; I found them both at 103; the weather was frosty.

December 27. I examined the air at Senlis, and that which I had gathered in the middle of the day on the road; both were at 102½; the weather continued frosty.

December 29. Being at Paris, I found the air in that capital at 103; the weather frosty.

1780, January 8, the air of Paris was at 100; the weather very frosty.

January 13. The air of Paris was 98; it froze very hard.

This paper being already too long, I will only add some few deductions, which seem to follow of themselves from the above related experiments.

It appears from these experiments, that the air at sea and close to it is in general purer and fitter for animal life than the air on the land, though it seems to be subject to the same inconstancy in its degree of purity with that of the land; so that we may now with more confidence send our patients, labouring under consumptive disorders, to the sea, or at least to places situated close to the sea, which have no marshes in their neighbourhood.

It

It ſeems alſo probable, that the air will be found in general much purer far from the land than near the ſhore, the former being never ſubject to be mixed with land air.

It appears alſo, that the air in froſty weather is in general wholeſomer than it is in winter when it does not freeze; and that uncommon warm weather, happening in the winter ſeaſon, is apt to render the atmoſphere very unwholeſome: the reaſon of which I apprehend to be, that the froſt totally checks that general tendency to corruption, which being revived by warmth again increaſes the infection of the common air, which at that time is ſo much the greater, becauſe the plants (which are deprived of their leaves in winter) have no power in them to counteract it.

It ſeems alſo probable, that thoſe countries which are, by their local ſituation, expoſed to noxious exhalations, are in general much wholeſomer in the winter; and that it is much ſafer to croſs ſuch countries in ſummer time when it is windy weather than in a calm, &c.

How far theſe deductions are founded upon experience may appear by applying them to ſuch places as they may be found to have a relation to.

My old friend Dr. DAMMAN, an eminent phyſician and profeſſor royal in midwifry at Ghent, told me, that when

when he was formerly a practitioner at Oſtend, during ſeven years, he found the people there remarkably healthy; that nothing was rarer there than to ſee a patient labouring under a conſumption or aſthma, a malignant, putrid, or ſpotted fever; that the diſeaſe to which they are the moſt ſubject, is a regular intermittent fever in autumn, when ſudden tranſitions from hot to cold weather happen.

People are in general very healthy at Gibraltar, though there are very few trees near that place; which, I think, is owing to the purity of the air, ariſing from the neighbourhood of the ſea.

Moſt ſmall iſlands are very healthy.

At Malta people are little ſubject to diſeaſes, and live to a very advanced age.

XXIV. *The principal Properties of the Engine for turning Ovals in Wood or Metal, and of the Inſtrument for drawing Ovals upon Paper, demonſtrated. By the Rev. Mr.* Ludlam, *Vicar of* Norton, *near* Leiceſter; *communicated by the Aſtronomer Royal.*

Read May 4, 1780.

THE inſtrument for drawing ovals upon paper or board is ſo common, that a particular deſcription of it is needleſs. It is much in uſe among the joiners, and called by them *the trammels*. One part of it conſiſts of a croſs with two grooves at right angles: the other is a beam carrying two pins which ſlide in thoſe grooves, and alſo the deſcribing pencil; we ſhall diſtinguiſh theſe two parts by the names of the *croſs* and the *beam*.

It is very well known, that all the engines for turning ovals are conſtructed on the ſame principles with the trammels; the only difference is, that in the trammels the board is at reſt, and the pencil moves upon it; in the turning engine, the tool (which ſupplies the place of the pencil) is at reſt, and the board moves againſt it.

Let

Let A*a* and B*b* (fig. 1.) be two indefinite lines, interſecting each other at right angles in C. Let LSM be the beam, or a rigid right line, in which aſſume two fixed points L and S at pleaſure. If the fixed point L be kept always ſliding upon the line BC*b*, and the other point S always ſliding upon the line AC*a*; I ſay then, that any point M in the line LS, or that line produced, will deſcribe an ellipſe.

Biſect LS in E, and through C and E draw the indefinite right line CEH. Upon LS as a diameter with the center E deſcribe a ſemi-circle, and becauſe LCS is a right angle, it will paſs through C, and EC=EL. Through M draw MPH perpendicular to AC meeting CE produced in H; and becauſe MH is parallel to CL, the triangles MEH and CEL are ſimilar, and HE=ME, and HE+EC=ME+EL, or CH=LM. The point H therefore always falls in the circumference of the circle HAD*a* deſcribed with the center C and radius CH=LM. Now the ſimilar triangles CHP and SMP give CH : SM :: PH : PM. But when L arrives at C, then LM (=CH) coincides with CA; and when S arrives at C, then SM coincides with CB; therefore CA : CB :: PH : PM, and $CA^2 : CB^2 :: PH^2 : PM^2$, or $CA^2 : CB^2 :: AP \times P\alpha : PM^2$, which is the property of an ellipſe, whoſe firſt ſemi-axe is CA or LM, and ſecond ſemi-axe is CB=SM.

Produce PM till it meets the circle in N, and draw the radius CN; then PH=PN and CA : CB :: PN : PM. Again, becauſe PCH=PCN, therefore NCD=ECL=ELC and CN is parallel to LM, and CL=NM. Draw M*p* perpendicular to B*b*, cutting CN in *n*, and for the like reaſon C*n*=SM=CB, and CS=M*n*. While the point M deſcribes an oval, the point E deſcribes a circle whoſe center is C and radius CE=$\frac{1}{2}$SL.

To the ruler MEL (fig. 2. and 3.) fix another ruler or right line *m*EK paſſing through E, ſo that the ruler *m*EK may be carried about by the ruler MEL, keeping the angle ME*m* between the two rulers invariable. On *m*EK take EV=EK, and each = ES or EL, I ſay, the point V will deſcribe a right line *a*VC*α* paſſing through C, and making an angle *a*CS with CA, equal to half ME*m* the angle made by the two rulers; the point K will alſo deſcribe a right line *b*KC*β* paſſing through C, and making an angle *b*CL, with CL, alſo equal to half ME*m*.

On the center E (fig. 2. and 3.) and with the radius EC deſcribe a circle and it will paſs through the points S, V, C, L, K; draw the lines VC and KC, and the angles SEV, and SCV, both ſtand on the ſame arch SV; the former at the center E, the latter at the circumference C; therefore the former is double the latter. In like manner the angles KEL and KCL both ſtand on the ſame arch KL,

the former at the center, the latter at the circumference; therefore the former is double the latter. Now as this holds in every position of the rulers during their joint motion, it is manifeſt, the points V and K will each deſcribe right lines, namely, $ac\alpha$ and $bc\beta$, paſſing through c and making the angles acA and bcL ($=BC\beta$) each equal to half $ME\,m$.

Hence the lines $ac\alpha$ and $bc\beta$, traced by the points V and K are at right angles, and the ruler $mVEK$ moves exactly in the ſame manner as if it was guided by the the points V and K ſliding on the lines $ac\alpha$ and $bc\beta$, at right angles to each other; juſt in the ſame manner as the ruler MSL is guided by the points S and L, ſliding on the lines AC and BC. Therefore if any point m be aſſumed in the line KVm as a deſcribing point, the figure deſcribed will be an ellipſe, the poſition of whoſe principal axes are the lines $ac\alpha$ and $bc\beta$; the center of the ellipſe being ſtill in c as before. If mK is taken equal to MI, the ellipſe thus deſcribed by the point m will be the ſame with that deſcribed by the point M, only in another poſition: its greater ſemi-axis ac making an angle with AC, the greater ſemi-axis of the former ellipſe, equal to half MEm, the angle which the rulers or lines ME and mE make with each other.

Scholium. This propoſition is demonſtrated in SCHOOTEN's *Exercitationes*, &c. p. 305.; but he makes twelve caſes of it: had he made uſe of the 20th of the 3d El. they might have been all comprehended under one.

In the turning of ovals, the top of the *reſt* which ſupports the tool is always made to paſs through S and L (fig. 1.) the two centers round which the oval engine turns; and in this caſe the ruler or line MSEL repreſents the top of the *reſt*. If the tool be held on any part of the *reſt* between the work-man, and the neareſt center as at M, an oval will be turned having its longer axis A*a* (in one poſition of the work) coinciding with the top of the *reſt*. As the tool is removed towards S, the oval will grow narrower and at S become a right line. Beyond S towards E it will grow rounder, and at E become a circle; beyond E it will grow narrower, and at L become a right line at right angles to the right line deſcribed when the tool was at S. If the tool be removed beyond L, it will deſcribe an oval again, whoſe longer axis is at right angles to the longer axis of the oval firſt deſcribed when the tool was at M. It may be very convenient to mark the points S and L and alſo their middle point E on the top or face of the *reſt* that ſupports the tool. If any thing be interpoſed between the tool and the top of the *reſt* ſo as to raiſe the tool above the line paſſing through the centers

S and

S and L, an oval will yet be described, whose center will be the same with that of the oval first described when the tool was at M; but its principal axis will cross the principal axis of that oval (fig. 2. and 3.). Draw right lines both from M the old place of the tool, and from *m* the new place of the tool, to the point E marked on the *rest*. Half the angle which these two lines make with each other will be the angle which the principal axis of the new oval makes with the principal axis of the old one.

It is well known, that when the oval engine is set in order for working, there is a part which slides back, and is then fixed, which separates the two centers of motion and gives the eccentricity; for the difference between the first and second semi-axes will be just as much as the centers are thus separated: call the distance between the two centers E; let now the tool be fixed in any place, upon, above, or below the *rest*; call *m*E the distance of the tool from the middle point between the centers (marked E on the *rest*) D; and the greater semi-axis of the oval so described will be D+$\frac{1}{2}$E, and the lesser semi-axis D−$\frac{1}{2}$E; and thus both the form and position of the oval will be known. All workmen know the tool must never be raised above the place where it was at first held, and we

ſee the reaſon; it would deſtroy the oval firſt begun to be turned, and form a new one in a different poſition.

But there is another difficulty in turning ovals, eſpecially ſuch as have mouldings, as picture-frames, &c. The tool generally has all the mouldings formed upon it: now if it be laid flat upon the *reſt*, and the engine ſet to work; the mouldings will in ſome places croſs the plane of the tool (or the top of the *reſt*) at right angles (as in turning circles), in other places obliquely. This will make the ſeveral members of the mouldings *leaner* or ſmaller in one part of the work than another. Nor will the caſe be altered if the mouldings be turned ſeparately. Analogous to this, when an oval is drawn by the trammels, the line deſcribed by the pencil will not, as in a circle, be always at right angles to the beam of the trammels. The oval line ſo drawn will be at right angles to the deſcribing beam, only at the extremity of the two principal axes where the beam coincides with thoſe axes; in all other places the oval line and beam make an oblique angle. It may be proper therefore to enquire how much this angle deviates from a right angle. This we ſhall call the angle of *deviation*.

All things as in fig. 1. draw the tangents TM and TN, to the point M in the ellipſe and the point N in the circle correſponding to each other; and from the nature of the

ellipſe

ellipſe theſe tangents will meet each other in the axis CA produced. Draw MG perpendicular to TM, and GMS will be the angle of deviation ſought. I ſay, the angle MTN, between the tangents to correſponding points in the ellipſe and circumſcribing circle, is equal to the angle of deviation GMS.

For becauſe TNC is a right-angled triangle, and NP perpendicular to TC; therefore TNP=NCP=MSP, that is (in the triangles MTN and GMS) the angles TNM and MSG are equal. In like manner, becauſe TMG is a right-angled triangle and MP perpendicular to TG, therefore TMP=MGP, and (in the triangle MTN and GMS) the angles TMN and MGS are equal; therefore in the ſame triangles, the remaining angles MTN and GMS are alſo equal.

To compute the angle MTN, we have by trigonometry $TP^2+PM\times PN : TP :: MN :$ tan. MTN, radius being unity. Call now CA=t, CB=c, CP=x, PM=y, CA−CB (or $t-c$)=d, and we have $PN=\sqrt{tt-xx}$; alſo $CD : CB :: PN : PM=\sqrt{tt-xx}\times\frac{c}{t}$, whence $PM\times PN=\overline{tt-xx}\times\frac{c}{t}$. Again $CP : PN :: PN : PT$, whence $TP=\frac{tt-xx}{x}$. Laſtly, $CD : BD :: PN : MN=\sqrt{tt-xx}\times\frac{d}{t}$; whence the tangent of MTN the angle ſought is $\frac{dx\sqrt{tt-xx}}{ttt-axx}$: and this is a *maximum* when $\frac{ttt}{2t-d}$, or $\frac{tt}{t+c}=xx$, or when $\frac{ccc}{t+c}=yy$, or when CP and PM have ſuch a proportion that $CP^2 : PM^2 :: CA^3 : CB^3$.

Let

Let AMB*ab* (fig. 4.) be an ellipſe whoſe center is C; draw the circumſcribed and inſcribed circles as before; the former cutting the ſecond axis produced in D, the latter cutting the firſt axis in *d*, and the ſecond axis in *b*. On D*b* as a diameter deſcribe a circle cutting the firſt axis A*a* in Q, draw DQ and *b*Q. Set off CR=DQ, join DR, and draw DP at right angles cutting the firſt axis in P, draw PM an ordinate to that axis, and M will be the point in the oval line where the angle of deviation is greateſt. Otherwiſe, upon C*b* produced ſet off C*r*=*b*Q, join *dr*, and draw *dp* at right angles cutting the ſecond axis in *p*: draw *p*M an ordinate to that axis, and M will be the point where the angle of deviation is greateſt.

At the maximum (when $xx=\frac{ttt}{t+c}$) $PN^2=\frac{ttc}{t+c}$, $PM^2=\frac{ccc}{t+c}$, and $TP^2=\frac{tcc}{t+c}$: whence $TP^2=PM\times PN$. Alſo, PN, PM, TP, are to each other as CA, CB, and $\sqrt{CA\times CB}$, reſpectively. Therefore, $\frac{CA-CB}{\sqrt{Aa\times Bb}}$ is the tangent of MTN, radius being unity. Alſo $\sqrt{\frac{CA}{CB}}$ is the tangent of NTP; and MTP is the complement of NTP: therefore MTN is twice the exceſs of NTP, above 45°.

Fig. 1.

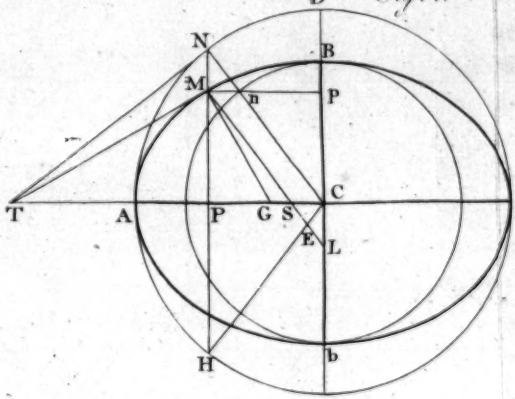


Fig. 2.

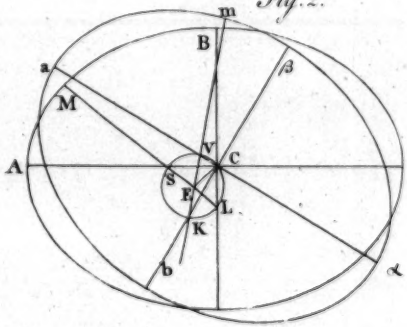


Fig. 1.

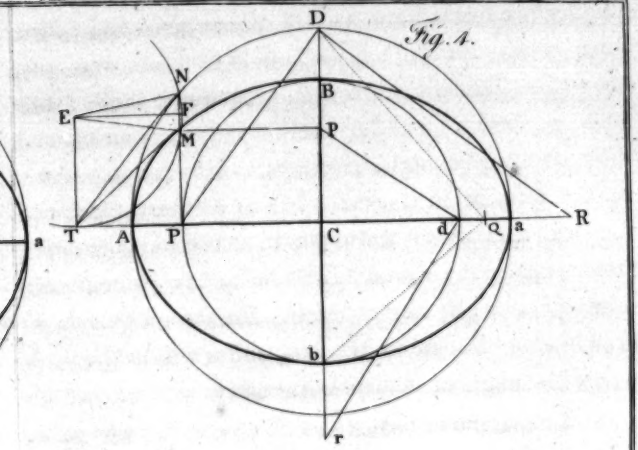
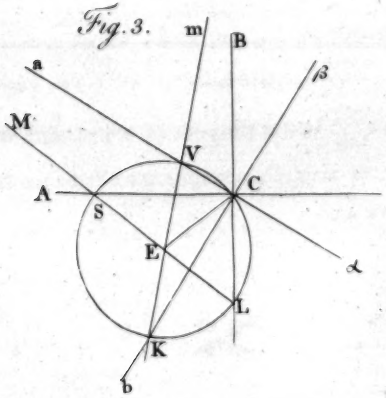


Fig. 3.



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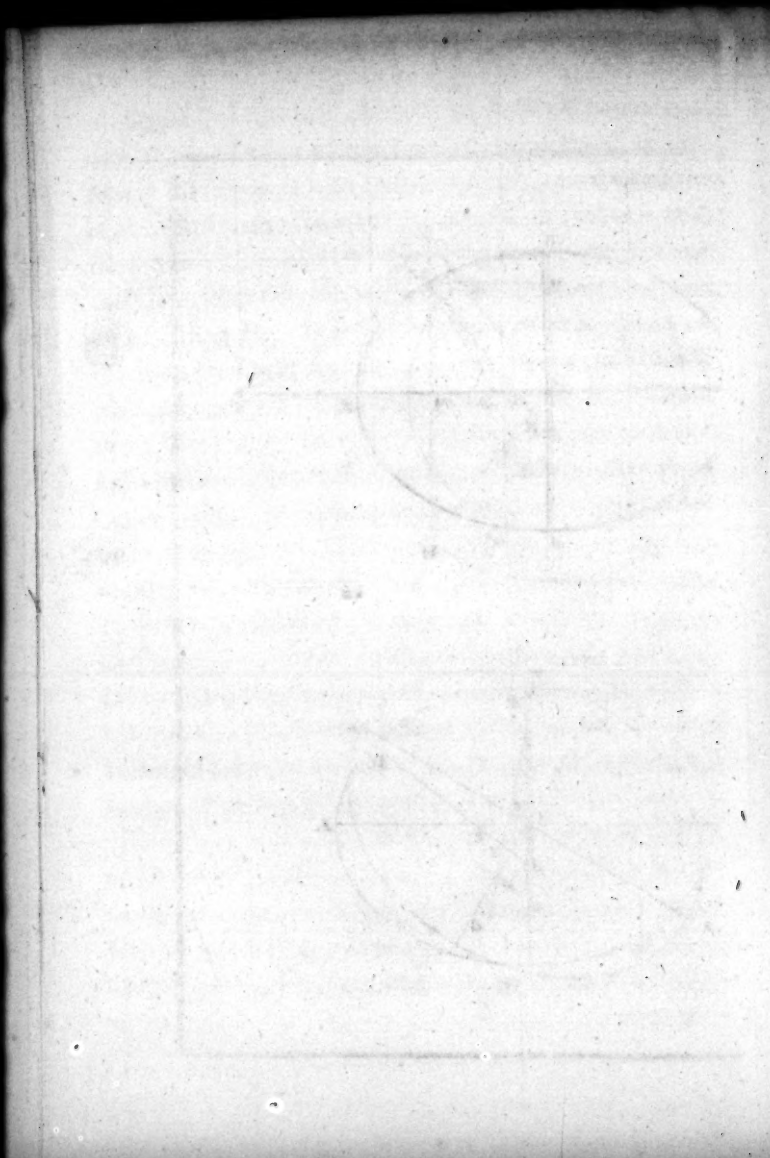

XXV. *Of Cubic Equations and Infinite Series.*

By Charles Hutton, *LL.D. F. R. S.*

Read June 1, 1780.

THE following pages are not to be underſtood as intended to contain a complete treatiſe on cubic equations, with all the methods of ſolution that have been delivered by other writers; but they are chiefly employed on the improvements of ſome properties that were before but partially known, with the diſcovery of ſeveral others which to me appear to be new and of no ſmall importance: for I have only ſlightly mentioned ſuch of the generally known properties as were neceſſary to the introduction or inveſtigation of the many curious conſequences herein deduced from them.

Art. 1. Every equation, whoſe terms are expreſſed in ſimple integral powers, has as many roots as there are dimenſions in the higheſt power. And when all the terms are brought to one ſide of the equation, and the

coefficient of the firſt term or higheſt power is +1, then the coefficient of the ſecond term is equal to the ſum of all the roots with contrary ſigns; the coefficient of the third term is equal to the ſum of all the products made by multiplying every two of the roots together; the coefficient of the fourth term, to the ſum of all the products ariſing from the multiplication of every three of the roots together; &c. and the laſt term, equal to the continual product of all the roots; the ſigns of all of them being ſuppoſed to be changed into the contrary ſigns before theſe multiplications are made. All this is evident from the generation of equations. And from theſe properties of the coefficients the following deductions are eaſily made.

2. If the ſigns of all the roots of an equation be changed, and another equation be generated from the ſame roots with the ſigns ſo changed; the terms of this laſt equation will have the ſame coefficients as the former, only the ſigns of all the even terms will be changed, but not thoſe of the odd terms: for the coefficients of the ſecond, fourth, and the other even terms, are made up of products conſiſting each of an odd number of factors; while thoſe of the third, fifth, and other odd terms are compoſed of products having an even number of factors: and the change of the ſigns of all the factors

produces a change in the ſign of the continual product of an odd number of factors, but no change in the ſign of that of an even number of factors. Wherefore, changing the ſigns of all the even terms, namely, the ſecond, fourth, &c. produces no alteration in the roots, but only in their ſigns, the poſitive roots being changed into negative, and the negative into poſitive. But by changing any or all the ſigns of the odd terms, the equation will no longer have the ſame roots as before, but will have new roots of very different magnitudes from thoſe of the former, unleſs the ſign of the firſt term or higheſt power is changed alſo; but this term is always to be ſuppoſed to remain poſitive.

3. It alſo follows, that when any term is wanting in an equation, or the coefficient of any term equal to 0, the ſum of the negative products in the coefficient of that term is equal to the ſum of the poſitive products in the ſame. And if it be the ſecond term which is wanting, then the equation has both negative and poſitive roots, and the ſum of the negative roots is equal to the ſum of the poſitive ones. But if it be the laſt term which is wanting, then one of the roots of the equation is equal to nothing. And hence ariſes a method of tranſforming any equation into another which ſhall want the ſecond term: and to this latter ſtate it will be proper to

transform every cubic equation before we attempt the reſolution of it.

4. Let therefore $x^3 + px = q$ be ſuch a cubic equation wanting the ſecond term, where p and q repreſent any numbers, poſitive or negative.

5. Now from the premiſes it follows, that this equation has three roots; that ſome are poſitive, and others negative; that two of them are of one affection, and are together equal to the third of a contrary affection, namely, either two negative roots, which are together equal to the other poſitive, or two poſitive roots equal to the third negative.

6. But the ſigns of the three roots are eaſily known from the ſign of the quantity q; the ſign of the greateſt root being the ſame with the ſign of q when this quantity is on the right-hand ſide of the equation, and the other two roots of the contrary ſign. For when q is on the ſame ſide of the equation with the other terms, it has been obſerved, that it is always equal to the continual product of all the roots with their ſigns changed; conſequently q is equal to the product of all the roots under their own ſigns, when that quantity is on the other or right-hand ſide of the equation: but the product of the two leſs roots is always poſitive, becauſe they are of the ſame affection, either both + or both −; and therefore

this product, drawn into the third or greateſt root, will generate another product equal to q, and of the ſame affection with this root.

7. But the roots of equations of the above form are not only poſitive, negative, or nothing, but ſometimes alſo imaginary. We have found that the greateſt root is poſitive when q is poſitive, and negative when it is negative; as alſo that one root is = to o when q is = o, and in this caſe the other two roots muſt be equal to each other, with contrary ſigns. But to diſcover the caſes in which the equation has imaginary roots, as well as many other properties of the equation, it will be proper to conſider the generation of it as follows.

8. The roots of equations becoming imaginary in pairs, the number of imaginary roots is always even; and therefore the cubic equation has either two imaginary roots, or none at all; and conſequently it has at leaſt one real root. Let that root be repreſented by r, which may be either poſitive or negative, and may be any one of the real roots, when none of them are imaginary: then ſince any one of the roots is equal to the ſum of the other two with their ſigns changed, the other two roots may be repreſented by $-\frac{1}{2}r \pm$ ſome other quantity, ſince the ſum of theſe two, with the ſigns changed, is $= r$. Now this ſupplemental quantity, which

is

is to be connected with $-\frac{1}{2}r$ by the signs + and − to compose the other two roots, will be a real quantity when those roots are real, but an imaginary one when they are imaginary, since the other part $(-\frac{1}{2}r)$ of those two roots is real by the hypothesis. Let this supplemental quantity be represented by e when it is real, or $e\sqrt{-1}$ or $\sqrt{-e^2}$ when it is imaginary: we shall use the quantity e in what follows for the real roots; and it is evident, that by changing e for $e\sqrt{-1}$, or e^2 for $-e^2$, that is, by barely changing the sign of e^2 wherever it is found, the expressions will become adapted to the imaginary roots. Hence then the three roots are represented by r, and $-\frac{1}{2}r+e$, and $-\frac{1}{2}r-e$; and consequently the three equations, from whose continual multiplication by one another the cubic equation is to be generated, will be $x-r=0$, and $x+\frac{1}{2}r-e=0$, and $x+\frac{1}{2}r+e=0$.

9. Let now these three equations be multiplied together, and there will be produced this general cubic equation wanting the second term, namely, $x^3 \overset{-\frac{3}{4}r^2}{-e^2} x - r.\overline{\frac{1}{4}r^2-e^2}=0$, or $x^3 \overset{-\frac{3}{4}r^2}{-e^2} x = r.\overline{\frac{1}{4}r^2-e^2}$, having three real roots; and if the sign of e^2 be changed from − to +, it will then represent all the cases which have only one real and two imaginary roots: and from the bare inspection of this equation the following properties are easily drawn.

10. First,

10. First, we hence find, that when the equation has three real roots, the sign of the second term is always $-$; for the coefficient of that term, or p is $= -\frac{3}{4}r^2 - e^2$, which is always negative when r and e are real quantities. And consequently when p is positive, the equation has two imaginary roots, since $-p$ includes all the cases of three real roots. But it does not therefore follow, that when p is negative, the three roots are always real; and indeed there are imaginary roots not only whenever p is positive, but sometimes also when p is negative: for since p is $= -\frac{3}{4}r^2 - e^2$ in all the cases of three real roots, it will be $p = -\frac{3}{4}r^2 + e^2$ for all the cases of two imaginary roots; and it is evident, that p will be either positive or negative, according as e^2 is greater or less than $\frac{3}{4}r^2$.

11. But to find the cases of $-p$ when the roots are all real, and when not, will require some farther consideration; and in order to that it must be observed, that e^2 ought to be positive and less than $\frac{3}{4}r^2$; but the limit between the cases of real and imaginary roots is when $e^2 = 0$, or $e = 0$; and then p becomes $= -\frac{3}{4}r^2$, and $q = \frac{1}{4}r^3$; consequently then $\overline{\frac{1}{3}p}^3 = \overline{\frac{1}{4}r^2}^3 = \frac{1}{64}r^6$, which is $= \overline{\frac{1}{2}q}^2 = \frac{1}{8}r^3\Big|^2 = \frac{1}{64}r^6$, that is, when e is $= 0$, then $\overline{\frac{1}{3}p}^3$ is $= \overline{\frac{1}{2}q}^2$, and consquently when $\overline{\frac{1}{3}p}^3$ is less than $\overline{\frac{1}{2}q}^2$, the equation has two imaginary roots, otherwise none, the sign of p being $-$.

Thus

Thus then we eaſily perceive in all caſes the nature of the roots as to real and imaginary; namely, partly from the ſign of p, and partly from the relation of p to q: for the equation has always two imaginary roots when p is poſitive; it has alſo two imaginary roots when p is negative, and $\frac{1}{3}p$ leſs than $\sqrt{\frac{1}{2}q}$; in the other caſe the roots are all real, namely, when p is negative and $\frac{1}{3}p$ either equal to or greater than $\sqrt{\frac{1}{2}q}$.

12. Moreover, when p is $= 0$, the equation has two imaginary roots; for this cannot happen but by $-e^2$ becoming $+e^2$, in the value of p, and $=$ to $\frac{3}{4}r^2$; and then $p = -\frac{3}{4}r^2 + e^2 = -\frac{3}{4}r^2 + \frac{3}{4}r^2 = 0$, and $q = r \cdot \sqrt{\frac{1}{4}r^2 + e^2} = r \cdot \sqrt{\frac{1}{4}r^2 + \frac{3}{4}r^2} = r \cdot r^2 = r^3$, and conſequently the above becomes barely $x^3 = r^3$, which therefore, beſides one real general equation root $x = r$, has alſo two imaginary roots.

13. Hence alſo it again appears, that the greateſt root is always of the ſame affection, in reſpect of poſitive and negative, with q on the right-hand ſide of the equation, they being either both poſitive or both negative together; and the other two roots of the contrary ſign. For if r be the greateſt root, then is $\frac{1}{2}r$ greater than e, and $\frac{1}{4}r^2$ greater than e^2, and $\frac{1}{4}r^2 - e^2$ always poſitive, and conſequently the product $r \cdot \sqrt{\frac{1}{4}r^2 - e^2}$, or q, will have the ſame ſign with r. But if r be one of the leſs roots, the contrary

trary of this will happen; for then $\frac{1}{2}r$ is leſs than e, and conſequently $\frac{1}{4}r^2$ leſs than e^2, and ſo $\frac{1}{4}r^2 - e^2$ a negative quantity, and therefore the product $r \cdot \sqrt{\frac{1}{4}r^2 - e^2}$, or q, will have the ſign contrary to that of r; that is, q and the leſs roots have different ſigns, and conſequently q and the greateſt root the ſame ſign, ſince the ſign of the greateſt root is always contrary to that of the other two roots.

14. Moreover, when q or $r \cdot \sqrt{\frac{1}{4}r^2 - e^2}$ is poſitive, then r denotes the greateſt root; for then $\frac{1}{4}r^2$ is greater than e^2, or $\frac{1}{2}r$ greater than e, and r greater than either $-\frac{1}{2}r + e$ or $-\frac{1}{2}r - e$. But when q or $r \cdot \sqrt{\frac{1}{4}r^2 - e^2}$ is negative, then r repreſents one of the other two roots in the equation; ſince then e is greater than $\frac{1}{2}r$, and $-\frac{1}{2}r - e$ greater than r. Laſtly, when q is between the poſitive and negative ſtates, or $q = 0$, then r ought to be neither the greateſt nor one of the leſs roots, if I may ſo ſpeak, that is, two of the roots are equal, and the third root $= 0$, ſince then $\frac{1}{4}r^2$ muſt be $= e^2$, or $\frac{1}{2}r = e$.

15. Hence it appears, that the ſign of p determines the nature of the roots as to real and imaginary, and the ſign of q determines the affection of the roots as to poſitive and negative. Let us illuſtrate theſe rules by a few examples.

16. The equation $x^3 - 9x = 10$ has all its three roots real, becauſe $p = -9$ is negative, and $\sqrt{\frac{1}{3}p|^3} = 3^3 = 27$ is

greater than $\overline{\frac{1}{2}q}^2$, which is $= 5^2 = 25$; and the greateſt of the roots is poſitive, becauſe $q = 10$ is poſitive; and the two leſs roots negative.

17. The equation $x^3 - 9x = -10$ has the ſame three real roots as the former, but with the contrary ſigns, the ſign of the greateſt root being now negative, becauſe $q = -10$ is negative.

18. But the equation $x^3 + 9x = \pm 10$ has only one real root and two imaginary roots, becauſe $p = 9$ is poſitive; and the ſign of the real root is + or − according as the ſign of q or 10 is + or −.

19. The equation $x^3 + 6x = \pm 10$ has alſo two imaginary roots, and one real root, which is + or − as it is $+10$ or -10, for the ſame reaſon as before.

20. The equation $x^3 - 6x = \pm 10$ has alſo two imaginary roots, becauſe $\overline{\frac{1}{3}p}^3 = 2^3 = 8$ is leſs than $\overline{\frac{1}{2}q}^2 = 5^2 = 25$.

21. But the equation $x^3 - 12x = \pm 16$ has all its roots real, becauſe $\overline{\frac{1}{3}p}^3 = 4^3 = 64$ is $= \overline{\frac{1}{2}q}^2 = 8^2 = 64$.

22. And the equation $x^3 + 12x = \pm 16$ has only one real root, becauſe $p = +12$ is poſitive.

23. Let us now conſider the other properties and relations of the roots ariſing from certain aſſumed relations between e and r, and from conſidering e either as real, imaginary, or nothing, that is e^2 as poſitive, negative, or nothing.

24. When

24. When e is a real quantity, the general equation is $x^3 - \frac{3}{4}r^2x = r \cdot \sqrt{\frac{1}{4}r^2 - e^2}$, and all the roots are real.

25. When e is imaginary, the general equation is $x^3 - \frac{3}{4}r^2x = r \cdot \sqrt{\frac{1}{4}r^2 + e^2}$, and two of the roots are imaginary.

26. When e is between theſe two ſtates, or $= 0$, the equation becomes $x^3 - \frac{3}{4}r^2x = \frac{1}{4}r^3$, and the root $r = \sqrt{\frac{4}{3}p} = \sqrt[3]{4q} = \frac{3q}{p}$; for in this caſe $p = \frac{3}{4}r^2$, and $q = \frac{1}{4}r^3$. Alſo the other two roots $-\frac{1}{2}r \pm e$ are each $= -\frac{1}{2}r$.

27. Aſſume now any general relation between the root r and the ſupplemental part e of the other two roots, as ſuppoſe $r^2 : e^2 :: 4 : n$, or $e^2 = \frac{n}{4}r^2$, or $e = \frac{1}{2}r\sqrt{n}$, where n repreſents either nothing or any quantity whether poſitive or negative, that is, poſitive when e and all the three roots are real, or negative when e and two of the roots are imaginary. Subſtitute now $\frac{n}{4}r^2$ inſtead of e^2 in the general equation $x^3 - \frac{3}{4}r^2x = r \cdot \sqrt{\frac{1}{4}r^2 - e^2}$, and that equation will become $x^3 - \frac{3+n}{4}r^2x = \frac{1-n}{4}r^3$. Here then $p = \frac{3+n}{4}r^2$, and $q = \frac{1-n}{4}r^3$, and conſequently the root $r = \sqrt{\frac{4p}{3+n}} = \sqrt[3]{\frac{4q}{1-n}} = \frac{n+3}{n-1} \cdot \frac{q}{p}$ expreſſed in three different ways. The other roots, the general values of which are $-\frac{1}{2}r \pm e$, become $-\frac{1}{2}r \pm \sqrt{\frac{n}{4}r^2} = -\frac{1}{2}r \pm \frac{1}{2}r\sqrt{n} = -\frac{1}{2}r \times 1 \pm \sqrt{n}$.

28. Hence then in an eaſy and general manner we can repreſent any form or caſe of the general equation, with all the circumſtances of the roots, by only taking, in theſe laſt formulæ, any particular number for n, either poſitive or negative, integral or fractional, &c. As if $n = 1$; then the equation becomes $x^3 - r^2x = \frac{0}{2}r^3$, or $= 0$, the value of $e = \frac{1}{2}r$, the root $r = \sqrt{p} = \sqrt[3]{\frac{4q}{0}} = \frac{4q}{0p}$, and the other two roots $= -\frac{2}{2}r.\ 1 \pm \sqrt{1} = -\frac{1}{2}r.\ 2$ and $-\frac{1}{2}r.\ 0 = -r$ and 0.

29. If $n = -1$, the equation will be $x^3 - \frac{1}{2}r^2x = \frac{1}{2}r^3$, the value of $e = \frac{1}{2}r\sqrt{-1}$, the root $r = \frac{q}{p} = \sqrt{2p} = \sqrt[3]{2q}$, and the other two roots $= -\frac{1}{2}r.\ 1 \pm \sqrt{1}$ imaginary.

30. And thus, by taking ſeveral different values of n, poſitive and negative, the various correſponding circumſtances and relations of the equation and roots will be exhibited as in the following table.

Forms

Forms of equat.	Values of r	Values of q.	Forms of the equation.	Values of the root r, viz. $r =$	Values of the two other roots viz.
	$+n$	$\frac{1}{2}r\sqrt{+n}$	$x^3 - \frac{n+3}{4}r^2x = -\frac{n-1}{4}r^3$	$\frac{n+1.q}{p} = \sqrt{\frac{4p}{n+3}} = \sqrt[3]{\frac{4}{n-1}}$	$-\frac{1}{2}r \times 1 \pm \sqrt{+n}$
1	$+12$	$\frac{1}{2}r\sqrt{+12}$	$x^3 - \frac{15}{4}r^2x = -\frac{11}{4}r^3$	$\frac{15q}{11p} = \sqrt{\frac{4p}{11}} = \sqrt[3]{\frac{4q}{11}}$	$1 \pm \sqrt{+12}$
2	$+11$	$\frac{1}{2}r\sqrt{+11}$	$x^3 - \frac{14}{4}r^2x = -\frac{10}{4}r^3$	$\frac{14q}{10p} = \sqrt{\frac{4p}{10}} = \sqrt[3]{\frac{4q}{10}}$	$1 \pm \sqrt{+11}$
3	$+10$	$\frac{1}{2}r\sqrt{+10}$	$x^3 - \frac{13}{4}r^2x = -\frac{9}{4}r^3$	$\frac{13q}{9p} = \sqrt{\frac{4p}{9}} = \sqrt[3]{\frac{4q}{9}}$	$1 \pm \sqrt{+10}$
4	$+9$	$\frac{1}{2}r\sqrt{+9}$	$x^3 - \frac{12}{4}r^2x = -\frac{8}{4}r^3$	$\frac{12q}{8p} = \sqrt{\frac{4p}{8}} = \sqrt[3]{\frac{4q}{8}}$	$1 \pm \sqrt{+9}$
5	$+8$	$\frac{1}{2}r\sqrt{+8}$	$x^3 - \frac{11}{4}r^2x = -\frac{7}{4}r^3$	$\frac{11q}{7p} = \sqrt{\frac{4p}{7}} = \sqrt[3]{\frac{4q}{7}}$	$1 \pm \sqrt{+8}$
6	$+7$	$\frac{1}{2}r\sqrt{+7}$	$x^3 - \frac{10}{4}r^2x = -\frac{6}{4}r^3$	$\frac{10q}{6p} = \sqrt{\frac{4p}{6}} = \sqrt[3]{\frac{4q}{6}}$	$1 \pm \sqrt{+7}$
7	$+6$	$\frac{1}{2}r\sqrt{+6}$	$x^3 - \frac{9}{4}r^2x = -\frac{5}{4}r^3$	$\frac{9q}{5p} = \sqrt{\frac{4p}{5}} = \sqrt[3]{\frac{4q}{5}}$	$1 \pm \sqrt{+6}$
8	$+5$	$\frac{1}{2}r\sqrt{+5}$	$x^3 - \frac{8}{4}r^2x = -\frac{4}{4}r^3$	$\frac{8q}{4p} = \sqrt{\frac{4p}{8}} = \sqrt[3]{\frac{4q}{4}}$	$1 \pm \sqrt{+5}$
9	$+4$	$\frac{1}{2}r\sqrt{+4}$	$x^3 - \frac{7}{4}r^2x = -\frac{3}{4}r^3$	$\frac{7q}{3p} = \sqrt{\frac{4p}{7}} = \sqrt[3]{\frac{4q}{3}}$	$1 \pm \sqrt{+4}$
10	$+3$	$\frac{1}{2}r\sqrt{+3}$	$x^3 - \frac{6}{4}r^2x = -\frac{2}{4}r^3$	$\frac{6q}{2p} = \sqrt{\frac{4p}{6}} = \sqrt[3]{\frac{4q}{2}}$	$\pm \sqrt{+3}$
11	$+2$	$\frac{1}{2}r\sqrt{+2}$	$x^3 - \frac{5}{4}r^2x = -\frac{1}{4}r^3$	$\frac{5q}{1p} = \sqrt{\frac{4p}{5}} = \sqrt[3]{\frac{4q}{1}}$	$\pm \sqrt{+2}$
12	$+1$	$\frac{1}{2}r\sqrt{+1}$	$x^3 - \frac{4}{4}r^2x = \mp\frac{0}{4}r^3$	$\frac{4q}{0p} = \sqrt{\frac{4p}{4}} = \sqrt[3]{\frac{4q}{0}}$	$\pm \sqrt{+1}$
13	± 0	$\frac{1}{2}r\sqrt{\pm 0}$	$x^3 - \frac{3}{4}r^2x = +\frac{1}{4}r^3$	$\frac{3q}{1p} = \sqrt{\frac{4p}{3}} = \sqrt[3]{\frac{4q}{1}}$	$\pm \sqrt{\pm 0}$
14	-1	$\frac{1}{2}r\sqrt{-1}$	$x^3 - \frac{2}{4}r^2x = +\frac{2}{4}r^3$	$\frac{2q}{2p} = \sqrt{\frac{4p}{2}} = \sqrt[3]{\frac{4q}{2}}$	$\pm \sqrt{-1}$
15	-2	$\frac{1}{2}r\sqrt{-2}$	$x^3 - \frac{1}{4}r^2x = +\frac{3}{4}r^3$	$\frac{1q}{3p} = \sqrt{\frac{4p}{1}} = \sqrt[3]{\frac{4q}{3}}$	$\pm \sqrt{-2}$
16	-3	$\frac{1}{2}r\sqrt{-3}$	$x^3 \mp \frac{0}{4}r^2x = +\frac{4}{4}r^3$	$\frac{0q}{4p} = \sqrt{\frac{4p}{0}} = \sqrt[3]{\frac{4q}{4}}$	$1 \pm \sqrt{-3}$
17	-4	$\frac{1}{2}r\sqrt{-4}$	$x^3 + \frac{1}{4}r^2x = +\frac{5}{4}r^3$	$\frac{1q}{5p} = \sqrt{\frac{4p}{1}} = \sqrt[3]{\frac{4q}{5}}$	$1 \pm \sqrt{-4}$
18	-5	$\frac{1}{2}r\sqrt{-5}$	$x^3 + \frac{2}{4}r^2x = +\frac{6}{4}r^3$	$\frac{2q}{6p} = \sqrt{\frac{3p}{2}} = \sqrt[3]{\frac{4q}{6}}$	$1 \pm \sqrt{-5}$
19	-6	$\frac{1}{2}r\sqrt{-6}$	$x^3 + \frac{3}{4}r^2x = +\frac{7}{4}r^3$	$\frac{3q}{7p} = \sqrt{\frac{4p}{3}} = \sqrt[3]{\frac{4q}{7}}$	$1 \pm \sqrt{-6}$
20	-7	$\frac{1}{2}r\sqrt{-7}$	$x^3 + \frac{4}{4}r^2x = +\frac{8}{4}r^3$	$\frac{4q}{8p} = \sqrt{\frac{4p}{4}} = \sqrt[3]{\frac{4q}{8}}$	$\pm \sqrt{-7}$
21	-8	$\frac{1}{2}\sqrt{-8}$	$x^3 + \frac{5}{4}r^2x = +\frac{9}{4}r^3$	$\frac{5q}{9p} = \sqrt{\frac{4p}{5}} = \sqrt[3]{\frac{4q}{9}}$	$1 \pm \sqrt{-8}$
22	-9	$\frac{1}{2}r\sqrt{-9}$	$x^3 + \frac{6}{4}r^2x = +\frac{10}{4}r^3$	$\frac{6q}{10p} = \sqrt{\frac{4p}{6}} = \sqrt[3]{\frac{4q}{10}}$	$1 \pm \sqrt{-9}$
23	-10	$\frac{1}{2}r\sqrt{-10}$	$x^3 + \frac{7}{4}r^2x = +\frac{11}{4}r^3$	$\frac{7q}{11p} = \sqrt{\frac{4p}{7}} = \sqrt[3]{\frac{4q}{11}}$	$1 \pm \sqrt{-10}$
24	-11	$\frac{1}{2}r\sqrt{-11}$	$x^3 + \frac{8}{4}r^2x = +\frac{12}{4}r^3$	$\frac{8q}{12p} = \sqrt{\frac{4p}{8}} = \sqrt[3]{\frac{4q}{12}}$	$1 \pm \sqrt{-11}$
25	-12	$\frac{1}{2}\sqrt{-12}$	$x^3 + \frac{9}{4}r^2x = +\frac{13}{4}r^3$	$\frac{9q}{13p} = \sqrt{\frac{4p}{9}} = \sqrt[3]{\frac{4q}{13}}$	$1 \pm \sqrt{-12}$
	$-n$	$\frac{1}{2}r\sqrt{-n}$	$x^3 + \frac{n-3}{4}r^2x = +\frac{n+1}{4}r^3$	$\frac{n-3.q}{n+1.p} = \sqrt{\frac{4p}{n+1}} = \sqrt[3]{\frac{4q}{n+1}}$	$1 \pm \sqrt{-n}$

31. From

31. From the bare inſpection of this table ſeveral uſeful and curious obſervations may be made. And firſt it appears, that when q is poſitive, as in all the forms after the 12th, r is the greateſt root; but when q is negative, or in all the caſes to the 12th, r is one of the leſs roots.

32. In all caſes before the 4th form r is the leaſt root, becauſe $\frac{\sqrt{10}-1}{2}$, or $\frac{\sqrt{11}-1}{2}$, &c. is always greater than 1; and in all ſuch forms $\overline{\frac{1}{2}q}^2$ is leſs than $\overline{\frac{1}{3}p}^3$; but the former approaches nearer and nearer to an equality with the latter till the 4th form, where $\overline{\frac{1}{2}q}^2$ is become $= \overline{\frac{1}{3}p}^3$, and r is then equal to one of the other roots, becauſe $\frac{\sqrt{9}-1}{2} = \frac{2}{2} = 1$.

33. From hence r becomes the middle root, and continues ſo to the 12th form, where it becomes equal to what has hitherto been the greateſt root, and the other root becomes at this place $= 0$; and $\overline{\frac{1}{2}q}^2$ has decreaſed from the 4th form all the way more and more in reſpect of $\overline{\frac{1}{3}p}^3$, till at this 12th form it has become $= 0$, or infinitely leſs than $\overline{\frac{1}{3}p}^3$.

34. From this place r becomes the greateſt root, the ſign of q changes to $+$, and $\overline{\frac{1}{2}q}^2$ again increaſes in reſpect of $\overline{\frac{1}{3}p}^3$, till at the 13th caſe it becomes again equal to it, and the two leſs roots equal to each other, like as at the 4th form.

35. From hence $\overline{\frac{1}{2}q}^3$ becomes greater than $\overline{\frac{1}{3}p}^3$, and increaſes more and more in reſpect of it, till at the 16th ſtep where p is $= 0$, or $\overline{\frac{1}{2}q}^3$ infinitely greater than $\overline{\frac{1}{3}p}^3$.

36. From this place the ſign of p becomes $+$, and $\overline{\frac{1}{2}q}^3$ continually decreaſes in reſpect of $\overline{\frac{1}{3}p}^3$ to infinity.

37. By help of this table we may find the roots of any cubic equation $x^3 \mp px = q$ whenever we can aſſign the relation between $\sqrt{p}$ and $\sqrt[3]{q}$. For ſince one root r is always $= \frac{n \pm 3 \cdot q}{n \mp 1 \cdot p} = \sqrt{\frac{4p}{n \pm 3}} = \sqrt[3]{\frac{4q}{n \mp 1}}$, and the other two roots $= -\frac{1}{2}r \cdot 1 \pm \sqrt{\pm n}$, it follows, that if from the equation $\sqrt{\frac{4p}{n \pm 3}} = \sqrt[3]{\frac{4q}{n \mp 1}}$, where the two denominators under the radicals differ by 4, we can aſſign the value of n, the above formula will give us the roots.

38. As if the equation be $x^3 - 18x = -27$. Here $p = 18$, and $q = 27$; then $\sqrt{\frac{4p}{8}} = \sqrt{\frac{p}{2}} = \sqrt{9} = 3$, and $\sqrt[3]{\frac{4q}{4}} = \sqrt[3]{27} = 3$ alſo; therefore $n + 3 = 8$, or $n - 1 = 4$, either of which gives $n = 5$: conſequently, $r = \frac{n + 3 \cdot q}{n - 1 \cdot p} = \frac{8q}{4p} = \frac{2q}{p} = \frac{54}{18} = 3$ is the middle root, becauſe $\frac{8q}{4p}$ is found between the 4th and 12th caſes, which are the limits of the middle roots: and $-\frac{1}{2}r \cdot 1 \pm \sqrt{n} = -\frac{3}{2} \cdot 1 \pm \sqrt{5} =$ 4·854102 and 1·854102 are the greateſt and leaſt roots. Or, theſe two roots may be alſo found in the ſame manner

manner from the table of forms, which contains all the roots of every equation, thus: by a few trials I find $\sqrt{\frac{4p}{20\cdot95}} = \sqrt[3]{\frac{4q}{10\cdot95}}$ nearly, and therefore $\frac{20\cdot95q}{10\cdot95p} = 1\cdot854$ is the leaſt root, becauſe here $n = 17\cdot95$ which lies far above the limit for the leaſt roots, which is at the fourth form, where n is $= 9$. And laſtly, $\sqrt{\frac{4p}{3\cdot0557}} = \sqrt[3]{\frac{4q}{\cdot9143}}$ nearly, and therefore, $\frac{3\cdot0557q}{\cdot9143p} = 4\cdot854$ is the greateſt root, becauſe $\frac{3\cdot0557q}{\cdot9143p}$ is found between the 12th and 13th forms, which are the limits between which lies the greateſt root of every equation that has all its roots real.

39. Again, let the equation be $x^3 + 2x = 12$. Here $p = 2$, and $q = 12$; hence $\sqrt{\frac{4p}{2}} = \sqrt{2p} = \sqrt{4} = 2$, and $\sqrt[3]{\frac{4q}{6}} = \sqrt[3]{\frac{2}{3}q} = \sqrt[3]{8} = 2$ alſo; therefore $n - 3 = 2$, or $n + 1 = 6$, either of which gives $n = 5$. Conſequently, $r = \frac{n-2\cdot q}{n+1\cdot p} = \frac{2q}{6p} = \frac{q}{3p} = \frac{12}{6} = 2$, and the other two roots are $-\frac{1}{2}r \cdot 1 \pm \sqrt{-n} = -1 \cdot 1 \pm \sqrt{-5} = -1 \mp \sqrt{-5}$.

40. But it is only by trials that we find out a proper value for n in ſuch caſes as theſe; and this is perhaps attended with no leſs trouble than the ſearching out one of the roots by trials from the original cubic equation itſelf. This method of finding the roots would indeed be effectual and ſatisfactory if we had a direct method of deter-

determining the value of n from the equation $\sqrt{\frac{4p}{n \pm 3}} = \sqrt[3]{\frac{4p}{n \mp 1}}$ by an equation under the 3d degree; but by reducing this equation out of radicals, there refults another cubic equation of no leſs difficulty to reſolve than the original one. We muſt therefore ſearch for other methods of determining the roots; and firſt it will be proper to treat of the rule which is called CARDAN's.

41. Let $x^3 + px = q$ be the general equation where p and q denote any given numbers with their ſigns, poſitive or negative. And let $z + y$ denote one of the roots of this equation, that is, let the root be divided into any two parts z and y. Hence then $x = z + y$; which value of x being ſubſtituted for it in the original equation $x^3 + px = q$, that equation will become $z^3 + 3z^2y + 3zy^2 + y^3 + p \cdot \overline{z + y} = q$, or $z^3 + y^3 + 3zy \cdot \overline{z + y} + p \cdot \overline{z + y} = q$. Now on introducing the two unknown quantities z and y, we ſuppoſed only one condition or equation, namely, $z + y = x$; we are therefore yet at liberty to aſſume any other poſſible condition we pleaſe: but this other condition ought to be ſuch as will make the equation reducible to a ſimple one, or to a quadratic, in order to obtain from it the value of z or y: and for this purpoſe there does not ſeem to be any other proper condition beſide

that which ſuppoſes $3xy$ to be $= -p$; and in conſequence of this ſuppoſition, the equation becomes barely $x^3 + y^3 = q$. Now from the ſquare of this equation let four times the cube of $xy = -\frac{1}{3}p$ be ſubtracted, and there will remain $x^6 - 2x^3y^3 + y^6 = q^2 + \frac{4}{27}p^3$, the ſquare root of which is $x^3 - y^3 = \sqrt{q^2 + \frac{4}{27}p^3}$; this laſt being added to, and ſubtracted from, the equation $x^3 + y^3 = q$, we have $\begin{cases} 2x^3 = q + \sqrt{q^2 + \frac{4}{27}p^3} = q + 2\sqrt{\frac{1}{4}q^2 + \frac{1}{27}p^3}, \\ 2y^3 = q - \sqrt{q^2 + \frac{4}{27}p^3} = q - 2\sqrt{\frac{1}{4}q^2 + \frac{1}{27}p^3}, \end{cases}$ hence dividing by 2, and extracting the cube roots, we have $\left\{\begin{array}{l} x = \sqrt[3]{\frac{1}{2}q + \sqrt{\frac{1}{4}q^2 + \frac{1}{27}p^3}} \times 1 \text{ or } \times \frac{1 \pm \sqrt{-3}}{2} \\ y = \sqrt[3]{\frac{1}{2}q - \sqrt{\frac{1}{4}q^2 + \frac{1}{27}p^3}} \times 1 \text{ or } \times \frac{1 \mp \sqrt{-3}}{2} \end{array}\right\}$ the three values of x and y; for every quantity has three different forms of the cube root, and the cube root of 1, is not only 1, but alſo $\frac{1 + \sqrt{-3}}{2}$ or $\frac{1 - \sqrt{-3}}{2}$. Hence then the three values of $x + y$ or x, or the three roots of the equation $x^3 + px = q$, are $\sqrt[3]{\frac{1}{2}q + \sqrt{\frac{1}{4}q^2 + \frac{1}{27}p^3}} \times 1$ or $\times -\frac{1 + \sqrt{-3}}{2}$ or $\times -\frac{1 - \sqrt{-3}}{2} + \sqrt[3]{\frac{1}{2}q - \sqrt{\frac{1}{4}q^2 + \frac{1}{27}p^3}} \times 1$ or $\times -\frac{1 - \sqrt{-3}}{2}$ or $\times -\frac{1 + \sqrt{-3}}{2}$, where the ſigns of $\sqrt{-3}$ muſt be oppoſite in the values of x and y, that is, when it is $\frac{1 \pm \sqrt{-3}}{2}$ in the one, it muſt be $\frac{1 \mp \sqrt{-3}}{2}$ in the other,

otherwiſe

otherwiſe their product xy will not be $= -\frac{1}{3}p$, as it ought to be.

42. Or if we put $a = \frac{1}{3}p$, and $b = \frac{1}{2}q$, the ſame three roots will be

$\sqrt[3]{b + \sqrt{b^2 + a^3}} + \sqrt[3]{b - \sqrt{b^2 + a^3}}$ = the 1ſt root or r,

$-\frac{1}{2}\sqrt[3]{b + \sqrt{b^2 + a^3}} \cdot 1 - \sqrt{-3} - \frac{1}{2}\sqrt[3]{b - \sqrt{b^2 + a^3}} \cdot 1 + \sqrt{-3}$ the 2d root,

$-\frac{1}{2}\sqrt[3]{b + \sqrt{b^2 + a^3}} \cdot 1 + \sqrt{-3} - \frac{1}{2}\sqrt[3]{b - \sqrt{b^2 + a^3}} \cdot 1 + \sqrt{-3}$ the 3d root.

43. Or again, the 1ſt root r being $\sqrt[3]{b + \sqrt{b^2 + a^3}} + \sqrt[3]{b - \sqrt{b^2 + a^3}}$, the other two will be

$-\frac{1}{2}r + \frac{\sqrt{-3}}{2}\sqrt[3]{b + \sqrt{b^2 + a^3}} - \frac{\sqrt{-3}}{2}\sqrt[3]{b - \sqrt{b^2 + a^3}}$ = the 2d root, and

$-\frac{1}{2}r - \frac{\sqrt{-3}}{2}\sqrt[3]{b + \sqrt{b^2 + a^3}} - \frac{\sqrt{-3}}{2}\sqrt[3]{b - \sqrt{b^2 + a^3}}$ = the 3d root.

44. Or, if we put $s = \sqrt[3]{b + \sqrt{b^2 + a^3}}$, and $d = \sqrt[3]{b - \sqrt{b^2 + a^3}}$, the roots will be

$s + d = r$ the 1ſt root,

$-\frac{s+d}{2} + \frac{s-d}{2}\sqrt{-3}$ = the 2d root,

$-\frac{s+d}{2} - \frac{s-d}{2}\sqrt{-3}$ = the 3d root.

45. The firſt of theſe roots x or $r = s + d = \sqrt[3]{b + \sqrt{b^2 + a^3}} + \sqrt[3]{b - \sqrt{b^2 + a^3}}$, is that which is called CARDAN's rule, by whom it was firſt publiſhed, but invented by FERREUS. And this is always a real root, though it is not

always the greateſt root as it has been commonly thought to be.

46. The firſt root $r = s + d = \sqrt[3]{b + \sqrt{b^2 + a^3}} + \sqrt[3]{b - \sqrt{b^2 + a^3}}$, although it be always a real quantity, yet often aſſumes an imaginary form when particular numbers are ſubſtituted inſtead of the letters a and b, or p and q. And this it is evident will happen whenever a is negative and a^3 greater than b^2, or $\overline{\frac{1}{3}p}^3$ greater than $\frac{1}{4}q^2$; for then $\sqrt{b^2 + a^3}$ becomes $\sqrt{b^2 - a^3} = \sqrt{\frac{1}{4}q^2 - \overline{\frac{1}{3}p}^3}$ the ſquare root of a negative quantity, which is imaginary. And this will evidently happen whenever the equation has three real roots, but at no time elſe, that is in all the firſt 13 caſes of the foregoing table, wherein $\overline{\frac{1}{3}p}^3$ is greater than $\overline{\frac{1}{2}q}^2$, and p negative; the 4th and 13th only excepted, when $\overline{\frac{1}{3}p}^3$ is $= \overline{\frac{1}{2}q}^2$, and therefore $\sqrt{b^2 - a^3} = 0$, and two of the roots become equal, but with contrary ſigns. This root can never aſſume an imaginary form when a or p is poſitive, nor yet when p is negative and $\overline{\frac{1}{2}q}^2$ greater than $\overline{\frac{1}{3}p}^3$; for in both theſe caſes the quantity $\sqrt{b^2 \pm a^3}$ is real, or the ſquare root of a poſitive quantity. And theſe take place after the firſt 13 caſes of the table of forms, that is, in all the caſes which have only one real root. So that this rule of CARDAN's always gives the root

root in an imaginary form when the equation has no imaginary roots, but in the form of a real quantity when it has imaginary roots.

47. It may, perhaps, ſeem wonderful that CARDAN's theorem ſhould thus exhibit the root of an equation under the form of an imaginary or impoſſible quantity always when the equation has no imaginary roots, but at no time elſe; and it may juſtly be demanded what can be the reaſon of ſo curious an accident. But this ſeeming paradox will be cleared up by the following conſideration. It is plain, that this circumſtance muſt have happened either through ſome impropriety in the manner of deducing the values of z and y from the two aſſumed equations $x = z + y$, and $zy = -\frac{1}{3}p$, or elſe by ſome impoſſibility in one of theſe two conditions themſelves; but, on examination, the deductions are found to be all fairly drawn, and the operations rightly performed. The true cauſe muſt therefore lie concealed in one of theſe two conditions $x = z + y$ and $zy = -\frac{1}{3}p$. In the firſt of them it cannot be, becauſe it only ſuppoſes that a quantity x can be divided into two parts z and y, which is evidently a poſſible ſuppoſition: it can therefore no where exiſt but in the latter, namely, $zy = -\frac{1}{3}p$. Now this ſuppoſition is this, that the product of the two parts z and y, into which the conſtant quantity x is divided, is

equal

equal to $\frac{1}{3}p$ with its ſign changed. But this may always take place when p is poſitive; for then $-\frac{1}{3}p$ will be negative, and two numbers, the one poſitive and the other negative, may always be taken ſuch that their product ſhall be equal to any negative number whatever, and yet their ſum be equal to a given quantity x; and this is done by taking the poſitive one as much greater than x as the other is negative; for thus it is evident the poſitive and negative numbers may be increaſed without end: wherefore there is no impoſſibility in the ſuppoſition when p is poſitive; and therefore then the formula ought to exhibit only real quantities, that is, in all the caſes after the 16th in the table of forms, as we have before found. But the ſame thing cannot always happen when p is negative, or $-\frac{1}{3}p = zy$ is poſitive: for that zy may be poſitive, the ſigns of the two factors z and y muſt be alike, either both + or both −, that is, both + when the ſign of x is +, or both − when that is −: but it is well known, that the greateſt product which can be made of the two parts into which a conſtant quantity x may be divided, is when the parts are equal to each other, or each equal $\frac{1}{2}x$, and therefore the greateſt product is equal to $\overline{\frac{1}{2}x}^2$ or $\frac{1}{4}x^2$: wherefore if $\frac{1}{4}x^2$ be equal to or greater than $-\frac{1}{3}p$, the condition which ſuppoſes that zy is $= -\frac{1}{3}p$, is poſſible, and the formula ought to expreſs the root by real quantities

tities only, otherwiſe not; but $\frac{1}{4}x^3$, or $\frac{1}{4}r^2$, which is the ſame thing, is always leſs than $-\frac{1}{3}p$ in the firſt thirteen caſes of the table of forms; and therefore in all theſe caſes, which are thoſe in which $\frac{1}{3}p^3$ is greater than $\frac{1}{2}q^2$, or all thoſe which have three real roots, the formula ought to exhibit the root with imaginary quantities, as we have before found to happen; the 4th and 5th caſes only excepted, in which $\frac{1}{3}p^3$ is $= \frac{1}{2}q^2$, and therefore the quantity $\sqrt{b^2 - a^3}$ vaniſhes, and two of the roots are equal.

48. Thus then the real cauſe of this circumſtance is made manifeſt, and it is found to be the neceſſary conſequence of the arbitrary hypotheſis which was made, which is found to be poſſible only in certain caſes. So that we cannot expect the formula to exhibit a real quantity in the other caſes, ſince an impoſſible hypotheſis muſt needs lead to an abſurd concluſion.

49. The other two roots $-\frac{s+d}{2} \mp \frac{s-d}{2}\sqrt{-3}$ in their general ſtate appear in an imaginary form; but on the ſubſtitution of numbers for the letters in any example, they come out real or imaginary quantities in thoſe caſes in which they ought to be ſuch. For s being $= g + \sqrt{\mp h}$, and $d = g - \sqrt{\mp h}$ according as the roots are all real or only one is ſuch; and $-\frac{s+d}{2} = -g = -\frac{1}{2}r$ always half the one real root, we ſhall have $\frac{s-d}{2} = \sqrt{\mp h}$ according to

the ſaid two caſes; and conſequently $\frac{1-d}{2}\sqrt{-3} = \sqrt{\pm 3b}$ a real or an imaginary quantity according as the roots are to be real or imaginary.

50. The firſt root r being found from the formula $\sqrt[3]{b+\sqrt{b^2+a^3}} + \sqrt[3]{b-\sqrt{b^2+a^3}}$, or by any other means, the other two roots may be exhibited in ſeveral other forms beſides the foregoing, as may be ſhewn in the following manner.

51. The equation being $x^3 + px = q$, and one root r, by ſubſtitution we have $r^3 + pr = q$, and, by ſubtracting, it is - - - $x^3 - r^3 + p.\overline{x-r} = 0$, and, dividing by $x-r$, it becomes $x^2 + rx + r^2 + p = 0$.

Or this ſame equation may be found by barely dividing $x^3 + px - q = 0$ by $x - r = 0$, for the quotient is $x^2 + rx + r^2 + p = 0$. And the reſolution of this quadratic equation gives $x = -\frac{1}{2}r \pm \sqrt{-p - \frac{3}{4}r^2} = -\frac{1}{2}r \pm \frac{1}{2}\sqrt{-4p - 3r^2}$ the other two roots. And from hence again it appears, that theſe two roots are always imaginary when p in the given equation is poſitive; as alſo when it is negative and leſs than $\frac{3}{4}r^2$; which again include all the caſes of the table of forms after the 13th.

52. Again, ſince $r^3 + pr = q$, therefore $r^2 + p = \frac{q}{r}$, and $r^2 = -p + \frac{q}{r}$, and $-3r^2 = 3p - \frac{3q}{r}$; which being

ſubſtituted in the above value of the two roots, they become $-\frac{1}{2}r \pm \frac{1}{2}\sqrt{-p - \frac{3q}{r}}$.

53. And again, if $-p$ be expelled from this laſt form by means of its value $r^2 - \frac{q}{r}$, the ſame two roots will be expreſſed by $-\frac{1}{2}r \pm \frac{1}{2}\sqrt{r^2 - \frac{4q}{r}} = -\frac{1}{2}r \times 1 \pm \sqrt{1 - \frac{4q}{r^3}}$.

54. And farther, if r^3 be expelled from this laſt form by means of its value $q - pr$, the ſame two roots will alſo become $-\frac{1}{2}r \times 1 \pm \sqrt{1 - \frac{4q}{q - pr}} = -\frac{1}{2}r \times 1 \pm \sqrt{\frac{pr + 3q}{pr - q}}$.

55. We might have derived the above forms in yet another manner thus. The firſt root being r, let the other two roots be v and w: then we ſhall have theſe two equations, namely, $v + w = -r$, and $vwr = q$, or $vw = \frac{q}{r}$; from the ſquare of the firſt of theſe ſubtract four times the laſt, ſo ſhall $v^2 - 2vw + w^2 = r^2 - \frac{4q}{r}$; the root of this is $v - w = \sqrt{r^2 - \frac{4q}{r}}$, which being added to, and taken from $v + w = -r$, and dividing by 2, we have $\left.\begin{matrix} v \\ w \end{matrix}\right\} = -\frac{1}{2}r \pm \frac{1}{2}\sqrt{r^2 - \frac{4q}{r}} = -\frac{1}{2}r \times 1 \pm \sqrt{1 - \frac{4q}{r^3}}$, the ſame with one of the formulæ above given; and then by ſubſtitution the others will be deduced.

56. To illuſtrate now the rules $x = y + \delta$, or $= \frac{y + \delta}{2} \pm \frac{y - \delta}{2}\sqrt{-3}$, by ſome examples; ſuppoſe the

given equation to be $x^3 - 36x = 91$. Here $p = -36$, $q = 91$, $a = \frac{1}{3}p = -12$, $b = \frac{91}{2}$; then $c = \sqrt{b^2 + a^3} = \sqrt{\frac{8281}{4} - 1728} = \sqrt{\frac{1369}{4}} = \frac{37}{2}$, $s = \sqrt[3]{b + c} = \sqrt[3]{\frac{91}{2} + \frac{37}{2}} = \sqrt[3]{64} = 4$, and $d = \sqrt[3]{b - c} = \sqrt[3]{\frac{91}{2} - \frac{37}{2}} = \sqrt[3]{27} = 3$. Consequently, $r = s + d = 4 + 3 = 7$ the first root; and $-\frac{s+d}{2} \pm \frac{s-d}{2}\sqrt{-3} = \frac{-7 \pm \sqrt{-3}}{2}$ the other two roots, which are imaginary.

57. Ex. 2. Let the equation be $x^3 + 30x = 117$. Here $a = \frac{1}{3}p = 10$, $b = \frac{1}{2}q = \frac{117}{2}$; then $c = \sqrt{b^2 + a^3} = \sqrt{\frac{13689}{4} + 1000} = \sqrt{\frac{17689}{4}} = \frac{133}{2}$, $s = \sqrt[3]{b + c} = \sqrt[3]{\frac{117}{2} + \frac{133}{2}} = \sqrt[3]{\frac{250}{2}} = \sqrt[3]{125} = 5$, and $d = \sqrt[3]{b - c} = \sqrt[3]{\frac{117}{2} - \frac{133}{2}} = \sqrt[3]{-\frac{16}{2}} = \sqrt[3]{-8} = -2$. Consequently, $r = s + d = 5 - 2 = 3$ the first root; and $-\frac{s+d}{2} \pm \frac{s-d}{2}\sqrt{-3} = \frac{-3 \pm 7\sqrt{-3}}{2}$ the other two roots, which are imaginary.

58. Ex. 3. If the equation be $x^3 + 18x = 6$, we shall have $a = 6$, and $b = 3$; then $c = \sqrt{b^2 + a^3} = \sqrt{9 + 216} = \sqrt{225} = 15$, $s = \sqrt[3]{b + c} = \sqrt[3]{3 + 15} = \sqrt[3]{18}$, and $d = \sqrt[3]{b - c} = \sqrt[3]{3 - 15} = \sqrt[3]{-12} = -\sqrt[3]{12}$. Therefore $r = s + d = \sqrt[3]{18} - \sqrt[3]{12} = .331313$ the first root; and $-\frac{s+d}{2} \pm \frac{s-d}{2}\sqrt{-3} = -\frac{\sqrt[3]{18} - \sqrt[3]{12}}{2} \pm \frac{\sqrt[3]{18} + \sqrt[3]{12}}{2}\sqrt{-3}$ the other two roots.

59. Ex. 4. In the equation $x^3 - 15x = 4$, we have $a = -5$, $b = 2$; hence $c = \sqrt{b^2 + a^3} = \sqrt{4 - 125} = \sqrt{-121}$

$\sqrt{-121} = 11\sqrt{-1}$, $s = \sqrt[3]{b + c} = \sqrt[3]{2 + 11\sqrt{-1}} = 2 + \sqrt{-1}$, and $d = \sqrt[3]{b - c} = \sqrt[3]{2 - 11\sqrt{-1}} = 2 - \sqrt{-1}$. Wherefore $r = s + d = 4$ the firſt root; and

$-\frac{s+d}{2} \mp \frac{s-d}{2}\sqrt{-3} = -2 \pm \sqrt{-1} \cdot \sqrt{-3} = -2 \pm \sqrt{3}$

the other two roots, which are alſo real.

60. Ex. 5. The equation $x^3 - 6x = 4$ gives $a = -2$, and $b = 2$; therefore $c = \sqrt{b^2 + a^3} = \sqrt{4 - 8} = \sqrt{-4} = 2\sqrt{-1}$, $s = \sqrt[3]{b + c} = \sqrt[3]{2 + 2\sqrt{-1}} = -1 + \sqrt{-1}$, and $d = \sqrt[3]{b - c} = \sqrt[3]{2 - 2\sqrt{-1}} = -1 - \sqrt{-1}$. And hence $r = s + d = -2$ the firſt root; and

$-\frac{s+d}{2} \pm \frac{s-d}{2}\sqrt{-3} = 1 \pm \sqrt{-1} \cdot \sqrt{-3} = 1 \pm \sqrt{3}$ which are the two extremes, or the greateſt and leaſt roots. So that in this example, CARDAN's rule gives the middle root.

61. Ex. 6. Let the equation be $x^3 - 9x = -10$. Then $a = -3$ and $b = -5$; ſo that $c = \sqrt{b^2 + a^3} = \sqrt{25 - 27} = \sqrt{-2}$, $s = \sqrt[3]{b + c} = \sqrt[3]{-5 + \sqrt{-2}} = 1 + \sqrt{-2}$, and $d = \sqrt[3]{b - c} = \sqrt[3]{-5 - \sqrt{-2}} = 1 - \sqrt{-2}$.

Hence $r = s + d = 2$ the middle root; and

$-\frac{s+d}{2} \pm \frac{s-d}{2}\sqrt{-3} = -1 \pm \sqrt{-2} \cdot \sqrt{-3} = -1 \pm \sqrt{6}$

the greateſt and leaſt roots.

62. Ex. 7. Take the equation $x^3 - 12x = 9$. Here $a = -4$, and $b = \frac{9}{2}$; therefore $c = \sqrt{b^2 + a^3} = \sqrt{\frac{81}{4} - 64}$

$= \sqrt{-\frac{121}{4}} = \frac{11}{2}\sqrt{-7}$, $s = \sqrt[3]{b+c} = \sqrt[3]{\frac{9}{2} + \frac{5}{2}\sqrt{-7}} = -\frac{3}{2} + \frac{1}{2}\sqrt{-7}$, and $d = \sqrt[3]{b-c} = \sqrt[3]{\frac{9}{2} - \frac{5}{2}\sqrt{-7}} = -\frac{3}{2} - \frac{1}{2}\sqrt{-7}$. Hence $r = s + d = -3$ the middle root; and $-\frac{s+d}{2} \pm \frac{s-d}{2}\sqrt{-3} = \frac{3}{2} \pm \frac{1}{2}\sqrt{-7} \cdot \sqrt{-3} = \frac{3 \pm \sqrt{21}}{2}$ the greateſt and leaſt roots.

63. Ex. 8. Again, from the equation $x^3 - 12x = -8\sqrt{2}$, we have $a = -4$, and $b = -4\sqrt{2}$; hence $c = \sqrt{b^2 + a^3} = \sqrt{32 - 64} = \sqrt{-32} = 4\sqrt{-2}$, $s = \sqrt[3]{b+c} = \sqrt[3]{-4\sqrt{2} + 4\sqrt{-2}} = \sqrt{2} + \sqrt{-2}$, and $d = \sqrt{2} - \sqrt{-2}$. So that $r = s + d = 2\sqrt{2}$ the middle root; and $-\frac{s+d}{2} \pm \frac{s-d}{2}\sqrt{-3} = -\sqrt{2} \pm \sqrt{-2} \cdot \sqrt{-3} = -\sqrt{2} \pm \sqrt{6} = -\sqrt{2} \cdot 1 \mp \sqrt{3}$ the greateſt and leaſt roots.

64. Ex. 9. But the equation $x^3 - 15x = 22$ gives $a = -5$, and $b = 11$; and therefore $c = \sqrt{b^2 + a^3} = \sqrt{121 - 125} = \sqrt{-4}$, $s = \sqrt[3]{b+c} = \sqrt[3]{11 + \sqrt{-4}} = 1 - \sqrt{-4}$, and $d = -1 + \sqrt{-4}$. Conſequently $r = s + d = -2$ the leaſt root; and $-\frac{s+d}{2} \pm \frac{s-d}{2}\sqrt{-3} = 1 \pm \sqrt{-4} \cdot \sqrt{-3} = 1 \pm \sqrt{12}$ the two greater roots.

65. Ex. 10. Laſtly, in the equation $x^3 - 15x = 20$, we have $a = -5$, and $b = 10$; conſequently $c = \sqrt{b^2 + a^3} = \sqrt{100 - 125} = \sqrt{-25} = 5\sqrt{-1}$, $s = \sqrt[3]{b+c} = \sqrt[3]{10 + 5\sqrt{-1}}$, and $d = \sqrt[3]{10 - 5\sqrt{-1}}$. Therefore

$r =$

$r = s + d = \sqrt[3]{10 + 5\sqrt{-1}} + \sqrt[3]{10 - 5\sqrt{-1}}$ = the firſt root; and $-\frac{s+d}{2} \pm \frac{s-d}{2}\sqrt{-3} = -\frac{\sqrt[3]{10+5\sqrt{-1}} + \sqrt[3]{10-5\sqrt{-1}}}{2}$

$\pm \frac{\sqrt[3]{10+5\sqrt{-1}} - \sqrt[3]{10-5\sqrt{-1}}}{2}\sqrt{-3}$ = the other two roots.

66. Hence it appears, that CARDAN's rule $s + d$ brings out ſometimes the greateſt root, ſometimes the middle root, and ſometimes the leaſt root.

Of the Roots by Infinite Series.

67. Another way of aſſigning the roots of a cubic equation, may be by infinite ſeries, derived from the foregoing formulæ, namely, $s + d$ and $-\frac{s+d}{2} \pm \frac{s-d}{2}\sqrt{-3}$, or $\sqrt[3]{b+c} + \sqrt[3]{b-c}$ and

$-\frac{1}{2} \times \sqrt[3]{b+c} + \sqrt[3]{b-c} \pm \frac{1}{2}\sqrt{-3} \times \sqrt[3]{b+c} - \sqrt[3]{b-c}$.

For by expanding $\sqrt[3]{b \pm c}$ in an infinite ſeries, we ſhall evidently have all the roots expreſſed in ſuch ſeries.

68. Now $s = \sqrt[3]{b+c} = \sqrt[3]{b} \times : 1 + \frac{c}{3b} - \frac{2c^2}{3 \cdot 6b^2} + \frac{2 \cdot 5c^3}{3 \cdot 6 \cdot 9b^3}$ &c.

and $d = \sqrt[3]{b-c} = \sqrt[3]{b} \times : 1 - \frac{c}{3b} - \frac{2c^2}{3 \cdot 6b^2} - \frac{2 \cdot 5c^3}{3 \cdot 6 \cdot 9b^3}$ &c.

Hence $s + d = 2\sqrt[3]{b} \times : 1 - \frac{2c^2}{3 \cdot 6b^2} - \frac{2 \cdot 5 \cdot 8c^4}{3 \cdot 6 \cdot 9 \cdot 12b^4}$ &c.

for the firſt root, as it was found by Mr. NICOLE, in the

Memoires

Memoires de l'Acad. 1738. Alſo

$$\left.\begin{array}{l} s-d=\frac{2c}{\sqrt[3]{b^2}}\times:\frac{1}{3}+\frac{2.5c^3}{3.6.9b^3}+\frac{2.5.8.11c^4}{3.6.9.12.15b^4}\ \&c.\ \text{Therefore,} \\ -\frac{s+d}{2} \\ \pm\frac{s-d}{2}\sqrt{-3} \end{array}\right\}=\left\{\begin{array}{l} -\sqrt[3]{b}\times:1-\frac{2c^2}{3.6b^2}-\frac{2.5.8c^4}{3.6.9.12b^4}\ \&c. \\ \pm\frac{c\sqrt{-3}}{\sqrt[3]{b^2}}\times:\frac{1}{3}+\frac{2.5c^3}{3.6.9b^6}+\frac{2.5.8.11c^4}{3.6.9.12.15b^8}\ \&c. \end{array}\right.$$

for the other two roots, which were given by CLAIRAUT, in his *Elemens d'Algebre*.

69. Hence again it appears, that when c^2 is poſitive, theſe two latter roots are imaginary; for then the factor $\frac{c\sqrt{-3}}{\sqrt[3]{b^2}}$ is imaginary. And that thoſe roots are real when this c^2 is negative; for then this factor becomes $\frac{c\sqrt{-1}\times\sqrt{-3}}{\sqrt[3]{b^2}}=\frac{c\sqrt{3}}{\sqrt[3]{b^2}}$, a real quantity. But in this laſt caſe, the ſign of every ſecond term in the two ſeries muſt be changed, namely, the ſigns of the terms containing the odd powers of the negative quantity c^2; for the ſeries contain the letters as adapted to the poſitive ſign only.

70. Theſe ſeries are proper for thoſe caſes only in which c^2 is not greater than b^2; for if c^2 were greater than b^2, they would all diverge, and be of no uſe: and the ſeries proper for the other caſes, namely, in which c^2 is greater than b^2, we ſhall give below.

71. That c^2 be leſs than b^2, or the foregoing ſeries be proper to be uſed, a or $\frac{1}{3}p$ muſt be a negative quantity; for if it be poſitive, then $c^2 = b^2 + a^3$ will be greater than b^2. But for this purpoſe a cannot be *any* negative quantity taken at pleaſure; for if it be ſo taken as that a^3 be greater than $2b^2$, then ſhall $-c^2 = a^3 - b^2$ be greater than b^2. And hence theſe ſeries converge only in ſome of the caſes of three real roots, and in ſome of thoſe that have only one real root, namely, from the 16th form to ſomewhere between the 12th and 13th forms in the general table Art. 30. when b is poſitive, and conſequently it includes ſome caſes both with and without imaginary roots. But that in all the caſes, the firſt ſeries $s + d = 2\sqrt[3]{b} \times : 1 - \frac{2c^2}{3.6b^2}$ &c. is the greateſt root, as will ſtill more fully appear by conſulting Art. 83.

72. Now, in the firſt place, when $a = 0$, or $c = b$, which is the limit, or 16th caſe in the table Art. 30, the equation being $x^3 = q = 2b$, then the only real root is $s = \sqrt[3]{b + c} = \sqrt[3]{2b} = \sqrt[3]{q} = \sqrt[3]{b} \times : 1 + \frac{1}{3} - \frac{2}{3.6} +$ &c. Hence alſo, dividing by $\sqrt[3]{b}$, we have

$$\sqrt[3]{2} = 1 + \frac{1}{3} - \frac{2}{3.6} + \frac{2.5}{3.6.9} - \frac{2.5.8}{3.6.9.12} \text{ &c.}$$

73. But in this caſe alſo the root is $s + d = 2\sqrt[3]{b} \times : 1 - \frac{2}{3.6} - \frac{2.5.8}{3.6.9.12}$ &c. And conſequently

conſequently this is equal to the former ſeries, or $2 \times : 1 - \frac{2}{3.6} - \frac{2.5.8}{3.6.9.12}$ &c. $= 1 + \frac{1}{3} + \frac{2}{3.6} + \frac{2.5}{3.6.9}$ &c. $= \sqrt[3]{2}$. Hence, by ſubtracting $1 - \frac{2}{3.6} - \frac{2.5.8}{3.6.9.12}$ &c. from both ſides, we have

$$1 - \frac{2}{3.6} - \frac{2.5.8}{3.6.9.12} \text{ \&c.} = \frac{1}{3} + \frac{2.5}{3.6.9} + \frac{2.5.8.11}{3.6.9.12.15} \text{ \&c.,}$$

which multiplied by $2\sqrt[3]{b}$ will alſo give the root of the ſame equation. And hence, adding $\frac{2}{3.6} + \frac{2.5.8}{3.6.9.12}$ &c. to both ſides of the laſt equation, we find that

$$1 \text{ is } = \frac{1}{3} + \frac{2}{3.6} + \frac{2.5}{3.6.9} + \frac{2.5.8}{3.6.9.12} + \frac{2.5.8.11}{3.6.9.12.15} \text{ \&c.}$$

Or, farther, multiplying by 3, and ſubtracting 1, we have

$$2 = \frac{2}{6} + \frac{2.5}{6.9} + \frac{2.5.8}{6.9.12} + \frac{2.5.8.11}{6.9.12.15} \text{ \&c.}$$

74. Alſo from $2 \times : 1 - \frac{2}{3.6} - \frac{2.5.8}{3.6.9.12}$ &c. $= \sqrt[3]{2}$ in the laſt article, we find $\frac{1}{2}\sqrt[3]{2} =$

$$\frac{1}{2} = \frac{2}{3.6} + \frac{2.5.8}{3.6.9.12} \text{ \&c.} = \frac{1}{3} + \frac{2.5}{3.6.9} + \frac{2.5.8.11}{3.6.9.12.15} \text{ \&c.}$$

75. In this caſe alſo, namely, $c = b$, the equation $d = \sqrt[3]{b} - c = \sqrt[3]{b} \times : 1 - \frac{c}{3b} - \frac{2c^2}{3.6b^2}$ &c. becomes

$$0 = \sqrt[3]{b} \times : 1 - \frac{1}{3} - \frac{2}{3.6} - \frac{2.5}{3.6.9} \text{ \&c.}$$

And hence, dividing by $\sqrt[3]{b}$, and adding, we have

$$1 = \frac{1}{3} + \frac{2}{3.6} + \frac{2.5}{3.6.9} + \frac{2.5.8}{3.6.9.12} \text{ \&c.}$$

the ſame as in the laſt article but one.

76. And

76. And by taking other values of b and c, or other relations between them, any number of infinite series may be assigned, whose sums will be given by the two equations $\sqrt[3]{b \pm c} = \sqrt[3]{b} \times : 1 \pm \frac{c}{3b} - \frac{2c^2}{3 \cdot 6b^2} \pm \frac{2 \cdot 5c^3}{3 \cdot 6 \cdot 9b^3}$ &c. And if b be very great in respect of c, the two first terms of the series will give the cube root true to many places of figures.

77. Hitherto is concerning one of the limits or extreme cases only, namely, when $c^2 = b^3$, or when the equation is $x^3 = q = 2b$. And it has been observed, that the first general series for the three roots converges in all the cases of the equation $x^3 - px = q$, or $x^3 - 3ax = 2b$, in which a^3 is not greater than $2b^2$. But a^3 may be any real quantity not greater than $2b^2$, and so it may be either less than, equal to, or greater than b^2.

78. When, in this equation, a^3 is less than b^2, then c^2 is positive, and less than b^3, and the first series gives the only real root without any change in the signs of the terms. And to this belongs all cases of the equation that can fall in between the 13th and 16th formulæ in the general table in Art. 30.

79. If a^3 be $= b^2$, then $c = 0$, and the three first series give $2\sqrt[3]{b} = \sqrt[3]{4q}$ for the greatest root, and $-\sqrt[3]{b}$ for each of the less roots. The same as at the 13th form in the general table Art. 30.

80. When a^3 is greater than b^3, c^3 will be negative, and then, changing the signs of the odd powers of c^2, the three general series will give the three roots of the equation, which will always be all real. In this class are two cases, namely, when c^2 is less than b^2, and when they are equal, which is the limit; for when c^2 becomes greater than b^2, the series diverge.

81. Now when a^3 is between b^3 and $2b^3$, then c^3 is negative and less than b^3, and the general series give all the three real roots by changing the sign of every other term.

82. And when $a^3 = 2b^3$, then $-c^3 = b^3$, and the three roots become thus:

$2\sqrt[3]{b} \times : 1 + \frac{2}{3.6} - \frac{2.5.8}{3.6.9.12} + \frac{2.5.8.11.14}{3.6.9.12.15.18}$ &c. the first or greatest root,

and $\left\{ \begin{array}{l} -\sqrt[3]{b} \times : 1 + \frac{2}{3.6} - \frac{2.5.8}{3.6.9.12} + \frac{2.5.8.11.14}{3.6.9.12.15.18} \text{ \&c.} \\ \mp \sqrt[3]{b} \times \sqrt{3} \times : \frac{1}{3} - \frac{2.5}{3.6.9} + \frac{2.5.8.11}{3.6.9.12.15} \text{ \&c.} \end{array} \right\}$

the two less roots.

83. The first of these 3 is the greatest root, because $\sqrt[3]{b} \times : 1 + \frac{2}{3.6} - \frac{2.5.8}{3.6.9.12}$ &c. is greater than $\sqrt[3]{b} \times \sqrt{3} \times : \frac{1}{3} - \frac{2.5}{3.6.9}$ &c. for $1 + \frac{2}{3.6}$ &c. is greater than 1, and $\sqrt{3} \times : \frac{1}{3} - \frac{2.5}{3.6.9}$ &c. $= \sqrt{\frac{1}{3}} \times : 1 - \frac{2.5}{6.9}$ &c. is

is leſs than 1. So that in general the firſt ſeries gives the greateſt of the three roots.

84. But it is evident, that this caſe agrees with the 10th form in the table Art. 30; in which the middle root r is found to be $\sqrt[3]{\frac{27}{2}q} = \sqrt[3]{2q} = -\sqrt[3]{4b} = -2\sqrt[3]{\frac{1}{2}b}$, and the two other, or greateſt and leaſt roots, are $-\frac{1}{2}r \times \overline{1 \pm \sqrt{3}} = \sqrt[3]{\frac{1}{2}b} \times \overline{1 \pm \sqrt{3}}$.

85. Hence by a compariſon of theſe two different forms of the ſame roots we find

$$\frac{\sqrt{3}+1}{2\sqrt[3]{2}} = 1 + \frac{2}{3.6} - \frac{2.5.8}{3.6.9.12} + \frac{2.5.8.11.14}{3.6.9.12.15.18} \text{ \&c.} = A,$$

and $$\frac{\sqrt{3}-1}{2\sqrt[3]{2}} = \frac{1}{3} - \frac{2.5}{3.6.9} + \frac{2.5.8.11}{3.6.9.12.15} - \text{\&c.} = B.$$

86. And by adding and ſubtracting theſe two, we find

$$\frac{\sqrt{3}}{\sqrt[3]{2}} = 1 + \frac{1}{3} + \frac{2}{3.6} - \frac{2.5}{3.6.9} - \frac{2.5.8}{3.6.9.12} + + - - \text{ \&c. and}$$

$$\frac{1}{\sqrt[3]{2}} = 1 - \frac{1}{3} + \frac{2}{3.6} + \frac{2.5}{3.6.9} - \frac{2.5.8}{3.6.9.12} - + + - - \text{ \&c.} = C.$$

87. Alſo, becauſe $\frac{\sqrt{3}+1}{2\sqrt[3]{2}} \times \frac{\sqrt{3}-1}{2\sqrt[3]{2}}$ is $= \frac{1}{2\sqrt[3]{4}}$, which is $= \frac{1}{2} \times \left[\frac{1}{\sqrt[3]{2}}\right]^2$; therefore the mean proportional between the two ſeries A and B, is to the ſeries C, as the ſide of a ſquare is to its diagonal.

88. Moreover, to and from the two ſeries A and B, adding and ſubtracting the two ſeries in Art. 74.

namely, $\frac{1}{2}\sqrt[3]{2}$ or $\frac{\sqrt[3]{2}}{2} = 1 - \frac{2}{3.6} - \frac{2.5.8}{3.6.9.12}$ &c. $= \frac{1}{3} + \frac{2.5}{3.6.9} + \frac{2.5.8.11}{3.6.9.12.15}$ &c. we obtain the 4 following series:

$$\frac{\sqrt{3}+\sqrt[3]{4}+1}{4\sqrt[3]{2}} = 1 - \frac{2.5.8}{3.6.9.12} - \frac{2.5.8.11.14.17.20}{3.6.9.12.15.18.21.24} \text{ &c.}$$

$$\frac{\sqrt{3}+\sqrt[3]{4}-1}{4\sqrt[3]{2}} = \frac{1}{3} + \frac{2.5.8.11}{3.6.9.12.15} + \frac{2.5.8.11.14.17.20.23}{3.6.9.12.15.18.21.24.27} \text{ &c.}$$

$$\frac{\sqrt{3}-\sqrt[3]{4}+1}{4\sqrt[3]{2}} = \frac{2}{3.6} + \frac{2.5.8.11.14}{3.6.9.12.15.18} \text{ &c.}$$

$$\frac{\sqrt{3}-\sqrt[3]{4}-1}{4\sqrt[3]{2}} = -\frac{2.5}{3.6.9} - \frac{2.5.8.11.14.17}{3.6.9.12.15.18.21} \text{ &c.}$$

89. It also appears, that the series

$$1 - \frac{1}{3} + \frac{2}{3.6} + \frac{2.5}{3.6.9} - \frac{2.5.8}{3.6.9.12} - \frac{2.5.8.11}{3.6.9.12.15} \text{ &c.}$$

is the reciprocal of the series

$$1 + \frac{1}{3} = \frac{2}{3.6} + \frac{2.5}{3.6.9} - \frac{2.5.8}{3.6.9.12} + \frac{2.5.8.11}{3.6.9.12.15} \text{ &c.}$$

where the signs of the former series are found by changing the signs of every other pair of terms in the latter; namely, omitting the first term, change the signs of the 2d and 3d terms, then passing over the 4th and 5th terms, change the signs of the 6th and 7th; and so on. For, by Art 86. the former of these series is equal to $\frac{1}{\sqrt[3]{4}}$; and, by Art. 72. the latter is equal to $\sqrt[3]{2}$.

90. Let

90. Let us now consider the cases in which c^3 is greater than b^2, which include all the cases not comprehended by the former, or in which c^3 is not greater than b^2. And this, it is evident, will happen both when a is positive and when negative; namely when a is any positive quantity whatever, or when it is any negative quantity, and a^3 greater than $2b^2$. And in these two classes, c^3 will be positive or negative, according as a is positive or negative.

91. Now the series in this class will be found the same way as in the last, by only writing here the letter c before the letter b; for then we shall have $s = \sqrt[3]{c+b}$, and $d = \sqrt[3]{-c+b} = -\sqrt[3]{c-b}$.

Then $s = \sqrt[3]{c+b} = \sqrt[3]{c} \times : 1 + \frac{b}{3c} - \frac{2b^2}{3.6c^2} + \frac{2.5b^3}{3.6.9c^3}$ &c.

and $d = -\sqrt[3]{c-b} = \sqrt[3]{c} \times : -1 + \frac{b}{3c} + \frac{2b^2}{3.6c^2} + \frac{2.5b^3}{3.6.9c^3}$ &c.

Hence $s + d = \frac{2b}{\sqrt[3]{c^2}} \times : \frac{1}{3} + \frac{2.5b^3}{3.6.9c^3} + \frac{2.5.8.11b^4}{3.6.9.12.15c^4}$ &c. $=$ the 1st root, and was given by CLAIRAUT. And

$$\left.\begin{array}{l} -\frac{s+d}{2} \\ \pm\frac{s-d}{2}\sqrt{-3} \end{array}\right\} = \left\{\begin{array}{l} \frac{-b}{\sqrt[3]{c^2}} \times : \frac{1}{3} + \frac{2.5b^3}{3.6.9c^3} + \frac{2.5.8.11b^4}{3.6.9.12.15c^4} \text{ &c.} \\ \pm\sqrt[3]{c}.\sqrt{-3} \times : 1 - \frac{2b^2}{3.6c^3} - \frac{2.5.8b^4}{3.6.9.12c} \text{ &c.} \end{array}\right.$$

for the other two roots, which, I believe, are new.

92. Here it again appears, that when c^3 is positive, the two latter roots are imaginary; because then

$\sqrt[3]{c} \times \sqrt{-3}$ will be imaginary. But if c^2 be negative, thoſe roots will be both real; ſince $\sqrt[3]{c} \times \sqrt{-3}$ then becomes $\sqrt[3]{c} \cdot \sqrt{-1} \times \sqrt{-3} = \sqrt[3]{c} \times -\sqrt{-1} \times \sqrt{-3} = -\sqrt[3]{c} \times \sqrt{3}$. The ſigns prefixed to the terms as above, take place when c^2 is poſitive; but when c^2 ſhall be negative, the ſigns of the terms containing the odd powers of it muſt be changed. And theſe ſeries include all the caſes in which the former ones failed by not converging. So that between them they comprehend all the caſes of the general cubic equation $x^3 \pm px = q$, as they each reciprocally converge when the other diverges, but in no other caſe, except in the common claſs, in which c is $= b$, which happens at the two limits, namely, either when a is $= 0$, or when $-a^3 = 2b^2$: and then they both give the ſame roots. But in the other caſes they give the contrary roots; namely, when c is leſs than b, the firſt ſeries gives the greateſt root; and when c is greater than b, the latter ſeries gives the leaſt root.

93. Now when a is any poſitive quantity, the firſt of theſe ſeries gives the only real root, without any change in the ſigns of the terms; the other two being imaginary. And this includes all the caſes after the 16th in the table in Art. 30.

94. When a is $= 0$, or the limit between poſitive and negative, as in the 16th form in Art. 30. then is $c = b$,

and the only real root, or the firſt ſeries, becomes $2\sqrt[3]{b} \times : \frac{1}{3} + \frac{2.5}{3.6.9} +$ &c. which is the ſame root as was before found in Art. 73. So that in this 116th caſe, both this ſeries and the ſeries in Art. 67. converge, and give the ſame and only real root.

95. When a becomes negative, then c^2 becomes negative, and the roots all real. But in this caſe the ſeries only begins to converge when $-a^3 = 2b^2$, for then $-c^2$ becomes $= b^2$, and then, making the proper change in the ſigns of the terms, the three roots become

1ſt. $-2\sqrt[3]{b} \times : \frac{1}{3} - \frac{2.5}{3.6.9} + \frac{2.5.8.11}{3.6.9.12.15}$ &c. the leaſt root, and

$$\begin{cases} +\sqrt[3]{b} \times : \frac{1}{3} - \frac{2.5}{3.6.9} + \frac{2.5.8.11}{3.6.9.12.15} \text{ &c.} \\ \pm \sqrt[3]{b} . \sqrt{3} \times : 1 + \frac{2}{3.6} - \frac{2.5.8}{3.6.9.12} + \frac{2.5.8.11.14}{3.6.9.12.15.18} \text{ &c.} \end{cases}$$

the two greater roots.

96. I have here ſaid, that the firſt of theſe three roots is the leaſt of them. To prove which, I aſſert, that $\sqrt{3} \times : 1 + \frac{2}{3.6} - \frac{2.5.8}{3.6.9.12}$ &c. is greater than 3 times $\frac{1}{3} - \frac{2.5}{3.6.9}$ &c. for $3 \times : \frac{1}{3} - \frac{2.5}{3.6.9}$ &c. $= 1 - \frac{2.5}{6.9}$ &c. is leſs than 1, whereas $1 + \frac{2}{3.6}$ &c. is greater than 1. Conſequently, the leſs of the two latter roots, namely, $\sqrt[3]{b}$

$\sqrt[3]{b} \cdot \sqrt{3} \times : 1 + \frac{2}{3.6}$ &c. $- \sqrt[3]{b} \times : \frac{1}{3} - \frac{2.5}{3.6.9}$ &c. is greater than the first root $2\sqrt[3]{b} \times : \frac{1}{3} - \frac{2.5}{3.6.9}$ &c. That is to say, here the first is the least of the three roots, while in the other class of series the first is the greatest root.

97. Hence, comparing the value of any one of the roots here found, with the value of the same root as found in Art. 82, we obtain the relation between the two series that are concerned in them, namely, that the series $1 + \frac{2}{3.6} - \frac{2.5.8}{3.6.9.12} + \frac{2.5.8.11.14}{3.6.9.12.15.18}$ &c. is to the series $\frac{1}{3} - \frac{2.5}{3.6.9} + \frac{2.5.8.11}{3.6.9.12.15} - \frac{2.5.8.11.14.17}{3.6.9.12.15.18.21}$ &c. as $\sqrt{3} + 1$ is to $\sqrt{3} - 1$, or as $2 + \sqrt{3}$ to 1, or as 1 to $2 - \sqrt{3}$, which are all equal to the same ratio. And the same thing appears from Art. 85.

98. When $-a^3$ becomes greater than $2b^2$, or a^6 is greater than b^2, and, by the proper change in the signs, the series for the roots in all cases of this kind become

1st. $\frac{-2b}{\sqrt{c}} \times : \frac{1}{3} - \frac{2.5b^2}{3.6.9c^2} + \frac{2.5.8.11b^4}{3.6.9.12.15c^4}$ &c. the least root,

and $\left\{ \begin{array}{l} + \frac{b}{\sqrt{c}} \times : \frac{1}{3} - \frac{2.5b^2}{3.6.9c^2} + \frac{2.5.8.11b^4}{3.6.9.12.15c^4} \text{ &c.} \\ \pm \sqrt[3]{c} \cdot \sqrt{3} \times : 1 + \frac{2b^2}{3.6c^2} - \frac{2.5.8b^4}{3.6.9.12c^4} \text{ &c.} \end{array} \right\}$ the two greater roots.

99. Let us now illustrate all the foregoing series for the roots of cubic equations, by finding by means of them

them the roots of the equations already treated of in Art. 56, &c.

100. And first in the equation $x^3 - 36x = 91$. Here $p = -36$, $q = 91$, $a = -12$, $b = 45\frac{1}{2}$, $c^3 = b^3 + a^3 = \overline{45\frac{1}{2}}^3 - 12^3 = 342\frac{1}{4}$, which being positive and less than b^2, this case belongs to the series

$2\sqrt[3]{b} \times : 1 - \frac{2c^2}{3.6b^2} - \frac{2.5.8c^4}{3.6.9.12b^4}$ – &c. in Art. 68.

Now $\frac{c^3}{b^3} = \frac{1369}{8281} = \overline{\frac{37}{91}}^3 = .1653182$. Then

A =	+	1.0000000
$B = \frac{2c^2}{3.6b^2} A$ =	–	.0183687
$C = \frac{5.8c^2}{9.12b^2} B$ =	–	11247
$D = \frac{11.14c^2}{15.18b^2} C$ =	–	1061
$E = \frac{17.20c^2}{21.24b^2} D$ =	–	118
$F = \frac{23.26c^2}{27.30b^2} E$ =	–	14
$G = \frac{29.32c^2}{33.36b^2} F$ =	–	2
sum of the terms =		.9803871
		2
		1.9607742 - log. 0.2924275
$\sqrt[3]{b} = \sqrt[3]{45.5}$ - - - - -		0.5526705
hence the only real root is 7 - - -		0.8450980

$= k$.

That is, $x = 7$ is $= 2\sqrt[3]{\frac{91}{2}} \times : 1 - \frac{2 \cdot 37^2}{3 \cdot 6 \cdot 91^2} - \frac{2 \cdot 5 \cdot 8 \cdot 37^4}{3 \cdot 6 \cdot 9 \cdot 12 \cdot 91^4}$ &c.

101. The other two roots are imaginary, and in Art. 56 they were found to be $= \frac{-7 \pm \sqrt{-3}}{2}$; but by means of the series in Art. 68, they are here found to be $\frac{-7}{2} \pm \frac{c\sqrt{-3}}{\sqrt[3]{b^3}} \times : \frac{1}{3} + \frac{2 \cdot 5 c^3}{3 \cdot 6 \cdot 9 b^6} +$ &c.

Consequently we obtain these following sums:

$$\frac{7}{2}\sqrt[3]{\frac{2}{91}} = 1 - \frac{2 \cdot 37^2}{3 \cdot 6 \cdot 91^2} - \frac{2 \cdot 5 \cdot 8 \cdot 37^4}{3 \cdot 6 \cdot 9 \cdot 12 \cdot 91^4} - \frac{2 \cdot 5 \cdot 8 \cdot 11 \cdot 14 \cdot 37^6}{3 \cdot 6 \cdot 9 \cdot 12 \cdot 15 \cdot 18 \cdot 91^6}$$ &c.,

$$\frac{1}{37}\sqrt[3]{\frac{91^2}{2^2}} = \frac{1}{3} + \frac{2 \cdot 5 \cdot 37^2}{3 \cdot 6 \cdot 9 \cdot 91^2} + \frac{2 \cdot 5 \cdot 8 \cdot 11 \cdot 37^4}{3 \cdot 6 \cdot 9 \cdot 12 \cdot 15 \cdot 91^4}$$ &c.

102. Ex. 2. In the equation $x^3 + 30x = 117$, we have $a = \frac{1}{3}p = 10$, $b = \frac{1}{2}q = \frac{117}{2} = 58\frac{1}{2}$, and $c^2 = b^2 + a^3 = \left(\frac{133}{2}\right)^2$, which being positive, and greater than b^2, the proper series for this is that in Art. 91, namely, $x = \frac{2b}{\sqrt[3]{c^2}} \times : \frac{1}{3} + \frac{2 \cdot 5 b^2}{3 \cdot 6 \cdot 9 c^2} + \frac{2 \cdot 5 \cdot 8 \cdot 11 b^4}{3 \cdot 6 \cdot 9 \cdot 12 \cdot 15 c^4} +$ &c.

Now $\frac{b^2}{c^2} = \left(\frac{117}{133}\right)^2 = .7738308$. Hence

$A = \frac{2}{3} \times = +333$

$B = \frac{2 \cdot 5 b^3}{6 \cdot 9 c^3} A = 48$ $\frac{2b}{\sqrt[3]{c^2}} = 7{\cdot}128$

$C = \frac{8 \cdot 11 b^3}{12 \cdot 15 c^3} B = 18$ series inverted 124

$D = \frac{14 \cdot 17 b^3}{18 \cdot 21 c^3} C = 9$ 2851

$E = \frac{20 \cdot 23 b^3}{24 \cdot 27 c^3} D = 5$ 142

7

$F = \frac{26 \cdot 29 b^3}{30 \cdot 33 c^3} E = 3$ the root $x = 3{\cdot}000$

$G = \frac{32 \cdot 35 b^3}{36 \cdot 39 c^3} F = 2$

$H = \frac{38 \cdot 41 b^3}{42 \cdot 45 c^3} G = 1$

$I = \frac{44 \cdot 47 b^3}{48 \cdot 51 c^3} H = 1$

$K = \frac{50 \cdot 53 b^3}{54 \cdot 57 c^3} I = 1$

sum of the terms = ·421

That is, $x = 3 = \frac{2 \cdot 117}{\sqrt[3]{2 \cdot 133^2}} \times : \frac{1}{3} + \frac{2 \cdot 5 \cdot 117^2}{3 \cdot 6 \cdot 9 \cdot 133^2} + \&c.$

103. By the other series in the same article the two imaginary roots come out

$= -\frac{3}{2} \pm \sqrt[3]{c} \cdot \sqrt{-3} \times : 1 - \frac{2b^3}{3 \cdot 6c^3} - \&c.$ which were before found in Art. 57 to be $-\frac{3}{2} \pm \frac{7}{2}\sqrt{-3}$. Consequently

$\frac{7}{2}\sqrt[3]{\frac{2}{133}} = 1 - \frac{2 \cdot 117^2}{3 \cdot 6 \cdot 133^2} - \frac{2 \cdot 5 \cdot 8 \cdot 117^4}{3 \cdot 6 \cdot 9 \cdot 12 \cdot 133^4} - \frac{2 \cdot 5 \cdot 8 \cdot 11 \cdot 14 \cdot 117^6}{3 \cdot 6 \cdot 9 \cdot 12 \cdot 15 \cdot 18 \cdot 133^6} \&c.$

$\frac{1}{39}\sqrt[3]{\frac{133^2}{2^6}} = \frac{1}{3} + \frac{2 \cdot 5 \cdot 117^2}{3 \cdot 6 \cdot 9 \cdot 133^2} + \frac{2 \cdot 5 \cdot 8 \cdot 11 \cdot 117^4}{3 \cdot 6 \cdot 9 \cdot 12 \cdot 15 \cdot 133^4} + \&c.$

104. Ex. 3. In the equation $x^3 + 18x = 6$, we have $a = 6$, $b = 3$, $c = \sqrt{9 + 216} = \sqrt{225} = 15$, real and greater than b, and therefore this case belongs to the same series as the last example. Now $\frac{b^2}{c^2} = \frac{9}{225} = \frac{1}{25} = .04$, and $\frac{2b}{\sqrt[3]{c^2}} = \frac{6}{\sqrt[3]{225}} = \sqrt[3]{\frac{24}{25}} = \frac{1}{5}\sqrt[3]{120} = \sqrt[3]{.96}$. Then

$$A = \frac{1}{3} = .3333333$$
$$B = \frac{2 \cdot 5b^2}{6 \cdot 9c^2} A = .24692$$
$$C = \frac{8 \cdot 11b^2}{12 \cdot 15c^2} B = .483$$
$$D = \frac{14 \cdot 17b^2}{18 \cdot 21c^2} C = .12$$

$.3358520$ - log. $\bar{1}.5261480$

$\sqrt[3]{.96}$ - - - - $\bar{1}.9940904$

the root $x = .3313130$ - - $\bar{1}.5202384$

And then the two imaginary roots are

$$-\frac{.3313}{2} \pm \sqrt[3]{c} \cdot \sqrt{-3} \times : 1 - \frac{2b^2}{3 \cdot 6c^2} \text{ \&c.}$$

105. But, in Art. 58, these three roots were found to be $\sqrt[3]{18} - \sqrt[3]{12}$, and $-\frac{\sqrt[3]{18} - \sqrt[3]{12}}{2} \pm \frac{\sqrt[3]{18} + \sqrt[3]{12}}{2}\sqrt{-3}$. Consequently we have

$$\frac{\sqrt[3]{18} + \sqrt[3]{12}}{2\sqrt[3]{15}} = 1 - \frac{2}{3 \cdot 6 \cdot 25^2} - \frac{2 \cdot 5 \cdot 8}{3 \cdot 6 \cdot 9 \cdot 12 \cdot 25^4} - \text{ \&c.}$$

$$\frac{\sqrt[3]{18} - \sqrt[3]{12}}{2}\sqrt[3]{\frac{25}{3}} = \frac{1}{3} + \frac{2 \cdot 5}{3 \cdot 6 \cdot 9 \cdot 25^2} + \frac{2 \cdot 5 \cdot 8 \cdot 11}{3 \cdot 6 \cdot 9 \cdot 12 \cdot 15 \cdot 25^4} + \text{ \&c.}$$

106. Ex

106. Ex. 4. In the equation $x^3 - 15x = 4$, we have $a = -5$, $b = 2$, and $c = \sqrt{b^2 + a^3} = \sqrt{-121} = 11\sqrt{-1}$, imaginary and greater than b, which belongs to the ſame ſeries as the laſt 2 examples, but changing the ſign where the odd powers of the negative quantity c^2 is concerned, as in Art. 98.

Now $\frac{b^2}{c^2} = \frac{2^2}{11^2} = \frac{4}{121}$, and $\frac{2b}{\sqrt[3]{c^2}} = \frac{4}{\sqrt[3]{121}} = \sqrt[3]{\frac{64}{121}}$. Then

	+		—
$A = \frac{1}{3}$	$= \cdot3333333$	$B = \frac{2 \cdot 5b^2}{6 \cdot 9c^2} A$	$= \cdot0020406$
$C = \frac{8 \cdot 14b^2}{12 \cdot 15c^2} B =$	330	$D = \frac{14 \cdot 17b^2}{18 \cdot 21c^2} C =$	7
	$+ \cdot3333663$		$- \cdot0020413$
	$- \cdot0020413$		

the ſeries $= \cdot3313250$ - log. $\bar{1}.5202543$

$\sqrt[3]{\frac{64}{121}}$ - - - $\bar{2}.9077982$

the leaſt root $= -\cdot2679492$ - - $\bar{1}.4280525$

107. To find the other roots by this method, we muſt ſum the ſeries $\sqrt[3]{c} \cdot \sqrt{3} \times : 1 + \frac{2b^2}{3 \cdot 6c^2} -$ &c. And as the terms of it are found by multiplying the terms A, B, C, &c. of the former by $\frac{3}{1}$, $\frac{9}{5}$, $\frac{15}{11}$, $\frac{21}{17}$, &c. reſpectively, we ſhall therefore have

$\alpha =$

$\alpha = \frac{1}{1}$ A $= 1$

$\beta = \frac{9}{5}$ B $= 0{\cdot}0036731$ $\quad \gamma = \frac{15}{11}$ C $= -{\cdot}0000450$

$\delta = \frac{21}{17}$ D $= 8$

$+ 1{\cdot}0036739$

$- \quad 0000450$

series $= + 1{\cdot}0036289 \quad -$ log. $0{\cdot}0015732$

$\sqrt[3]{11} \quad - \quad - \quad - \quad 0{\cdot}3471309$

$\sqrt{3} \quad - \quad - \quad - \quad - \quad 0{\cdot}2385606$

$\pm 3{\cdot}8660254 \quad - \quad 0{\cdot}5872647$

$\frac{1}{3}$ the leaſt root with a contr. ſign $\} \quad + 0{\cdot}1339746$

ſum $+ 4{\cdot}0000000$ greateſt root

diff. $- 3{\cdot}7320508$ middle root.

108. But the ſame 3 roots, found in Art. 59, are alſo 4, and $-2 \pm \sqrt{3}$; which being compared with the ſeries in this example, we find

$$\frac{1+2\sqrt{3}}{2\sqrt[3]{11}} = 1 + \frac{2 \cdot 2^3}{3 \cdot 6 \cdot 11^2} - \frac{2 \cdot 5 \cdot 8 \cdot 2^4}{3 \cdot 6 \cdot 9 \cdot 12 \cdot 11^4} + \frac{2 \cdot 5 \cdot 8 \cdot 11 \cdot 14 \cdot 2^6}{3 \cdot 6 \cdot 9 \cdot 12 \cdot 15 \cdot 18 \cdot 11^6} \text{ \&c.}$$

$$\frac{2-\sqrt{3}}{4}\sqrt[3]{121} = \frac{1}{3} - \frac{2 \cdot 5 \cdot 2^3}{3 \cdot 6 \cdot 9 \cdot 11^2} + \frac{2 \cdot 5 \cdot 8 \cdot 11 \cdot 2^4}{3 \cdot 6 \cdot 9 \cdot 12 \cdot 15 \cdot 11^3} - \text{ \&c.}$$

109. Ex. 5. In the equation $x^3 - 6x = 4$, we have $a = -2$, $b = 2$, and $c^2 = b^2 + a^3 = 4 - 8 = -4$, which being negative, and $= b^2$, this caſe belongs to the ſeries either in Art. 82 or 95. The operation of ſumming the terms by them is here omitted, becauſe ſo much

room

room would be neceſſary to ſet down ſo great a number of terms, and as the properties ariſing from the ſeries in this caſe have already been noticed above. The 3 roots of this equation have been found in Art. 60 to be -2 and $1 \pm \sqrt{3}$.

110. Ex. 6. In the equation $x^3 - 9x = -10$, we have $a = -3$, $b = -5$, and $c^2 = 25 - 27 = -2$, which being negative and leſs than b^2, the general ſeries in Art. 68, with the neceſſary change of the ſigns, will give the 3 roots. Now $\frac{c^2}{b^2} = \frac{2}{25} = \frac{8}{100} = \cdot 08$, and $\sqrt[3]{b} = -\sqrt[3]{5}$, alſo $\frac{c\sqrt{-3}}{\sqrt[3]{b^2}} = \frac{\sqrt{6}}{\sqrt[3]{25}}$. Hence

$A = \quad = 1$

$B = \frac{2c^2}{3 \cdot 6b^2} A = 0{\cdot}0088889$

$D = \frac{11 \cdot 14c^2}{15 \cdot 18b^2} C = \quad 120$

$+ 1{\cdot}0089009$

$- 0{\cdot}0002641$

$+ 1{\cdot}0086368$

2

$2{\cdot}0172736$ - - log. $0{\cdot}3047649$

$\sqrt[3]{b} = \sqrt[3]{5}$ - - - - $0{\cdot}2329900$

the greateſt root $= -3{\cdot}44948974$ - $0{\cdot}5377549$

$C = \frac{5 \cdot 8c^2}{9 \cdot 12b^2} B = \cdot 0002634$

$E = \frac{17 \cdot 20c^2}{21 \cdot 24b^2} D = \quad 7$

$- \cdot 0002641$

111. Then

111. Then for the other roots, by multiplying the terms A, B, C, &c. of the former by $\frac{1}{3}$, $\frac{5}{9}$, $\frac{11}{15}$, &c. we have

$\alpha = \frac{1}{3}$ A = ·3333333	$\epsilon = \frac{5}{9}$ B = ·0049383
$\gamma = \frac{11}{15}$ C = 1931	$\delta = \frac{17}{21}$ D = 97
$\varepsilon = \frac{23}{27}$ E = 6	− ·0049480
+ ·3335270	
− ·0049480	
·3285790 - - - log. $\bar{1}$·5166398	
$\frac{\sqrt{6}}{\sqrt[3]{25}}$ - - - - - - $\bar{1}$·9230956	
the second series ± ·27525513 - - $\bar{1}$·4397354	
$\frac{1}{2}$ the greatest root + 1·72474487	
middle root 2·00000000	
least root 1·44948974	

112. But, by Art. 61, these 3 roots were found to be 2 and -1 ± 6; which being compared with the series belonging to this case, we find

$$\frac{\sqrt{6}+1}{2\sqrt[3]{5}} = 1 + \frac{2.2}{3.6.25} - \frac{2.5.8.2^3}{3.6.9.12.25^2} + \frac{2.5.8.11.14.2^5}{3.6.9.12.15.18.25^3} \text{ &c.}$$

$$\frac{\sqrt{6}-2}{4}\sqrt[3]{25} = \frac{1}{5} - \frac{2.5.2}{3.6.9.25} + \frac{2.5.8.11.2^3}{3.6.9.12.15.25^2} - \text{ &c.}$$

113. Ex. 7. In the equation $x^3 - 12x = 9$, we have $a = -4$, $b = \frac{9}{2}$, and $b^2 = \frac{81}{4} - 64 = -\frac{175}{4}$, which being negative, and greater than b^2, we shall have 3 real roots by the series in Art. 98.

Now $\frac{b^3}{c^3} = \frac{81}{175}$, $\frac{b}{\sqrt[3]{c^2}} = \frac{9}{\sqrt[3]{350}} = \sqrt[3]{\frac{729}{350}}$, and $\sqrt[3]{c} = \sqrt[6]{\frac{175}{4}} = \sqrt[6]{43{\cdot}75}$. Then

$A = \frac{1}{3}$	$= {\cdot}33333$	$B = \frac{2 \cdot 5b^3}{6 \cdot 9c^3} A$	$= {\cdot}02857$
$C = \frac{8 \cdot 11b^3}{12 \cdot 15c^3} B$	$= 647$	$D = \frac{14 \cdot 17b^3}{18 \cdot 21c^3} C$	$= 188$
$E = \frac{20 \cdot 23b^3}{24 \cdot 27c^3} D$	$= 62$	$F = \frac{26 \cdot 29b^3}{30 \cdot 33c^3} E$	$= 22$
$G = \frac{32 \cdot 35b^3}{36 \cdot 39c^3} F$	$= 8$	$H = \frac{38 \cdot 41b^3}{42 \cdot 45c^3} G$	$= 3$
$I = \frac{44 \cdot 47b^3}{48 \cdot 51c^3} H$	$= 1$	$K = \frac{50 \cdot 53b^3}{54 \cdot 57c^3} I$	$= 1$
	$+ {\cdot}34051$		$- {\cdot}03071$
	$- {\cdot}03071$		
	${\cdot}30980$		
	2		
	${\cdot}61960$	- - log.	$\bar{1}{\cdot}7921114$
	$\sqrt[3]{\frac{729}{350}}$	- - -	$0{\cdot}1062198$
the leaſt root	$= - {\cdot}79128$	- -	$\bar{1}{\cdot}8983312$

114. Then, ſince the terms of the latter ſeries are found by multiplying the terms of the former by the fractions $\frac{3}{1}$, $\frac{9}{5}$, $\frac{15}{11}$, $\frac{21}{17}$, &c. they will be thus:

$\alpha = \frac{1}{1}$ A = 1·00000 | $\gamma = \frac{15}{11}$ C = ·00882
$\epsilon = \frac{27}{23}$ B = 5143 | $\eta = \frac{39}{35}$ E = 73
$\vartheta = \frac{21}{17}$ D = 232 | $\iota = \frac{51}{47}$ G = 9
$\zeta = \frac{33}{29}$ F = 25 | I = 1
$\theta = \frac{45}{41}$ H = 4 | − ·00965

+ 1·05404
− 0·00965

1·04439 - - - log. 0·0188627
$\sqrt[6]{43·75}$ - - - - - - 0·2734963
$\sqrt{3}$ - - - - - - - 0·2385606

last ſeries ± 3·39564 - - 0·5309196
$-\frac{1}{2}$ the firſt + 0·39564

greateſt root + 3·79128
middle root − 3·00000

115. But, by Art. 62, theſe ſame 3 roots are, -3, and $\frac{3 \pm \sqrt{21}}{2}$; which being compared with the ſeries belonging to this caſe, we find

$$\frac{\sqrt{21}+9}{12\sqrt[3]{350}}\sqrt{6} = 1 + \frac{2.81}{3.6.175} - \frac{2.5.8.81^2}{3.6.9.12.175^2} + \&c.$$

$$\frac{\sqrt{21}-3}{36}\sqrt[3]{350} = \frac{1}{3} - \frac{2.5.81}{3.6.9.175} + \frac{2.5.8.11.81^2}{3.6.9.12.15.175^2} - \&c.$$

116. Ex. 8. In the equation $x^3 - 12x = -8\sqrt{2}$, we have $a = -4$, $b = -4\sqrt{2}$, and $c^2 = 32 - 64 = -32$, which being negative, and equal to b^2, the 3 roots will be

be found, by both the forms of ſeries, like as in Ex. 5, Art. 109; but the operation is here omitted for the ſame reaſons as were there given. The 3 roots of this equation were, in Art. 63, found to be $2\sqrt{2}$ and $-\sqrt{2}\pm\sqrt{6}$.

117. Ex. 9. In the equation $x^3-15x=22$, we have $a=-5$, $b=11$, and $c^2=121-125=-4$, which being negative, and leſs than b^2, the ſeries in Art. 68 give theſe 3 roots:

Greateſt root $= 2\sqrt[3]{11}\times:1+\frac{2c^2}{3.6b^2}-\frac{2.5.8c^4}{3.6.9.12b^4}$ &c.

The two leſs roots $\left\{\begin{array}{l}-\sqrt[3]{11}\times:1+\frac{2c^2}{3.6b^2}-\frac{2.5.8c^4}{3.6.9.12b^4}\text{ \&c.}\\ \pm\frac{2\sqrt{3}}{\sqrt[3]{121}}\times:\frac{1}{3}-\frac{2.5c^2}{3.6.9b^2}+\frac{2.5.8.11c^4}{3.6.9.12.15b^4}\text{ \&c.}\end{array}\right\}$ where $\frac{c^2}{b^2}=\frac{4}{121}$

Here

$A = 1\cdot0000000$

$B=\frac{2c^2}{3.6b^2}A$ $\quad 36731$

$D=\frac{11.14c^2}{15.18b^2}C$ $\quad 8$

$C=\frac{5.8c^2}{9.12b^2}B=-\cdot0000450$

$+1\cdot0036739$

$-0\cdot0000450$

$1\cdot0036289$

2

$2\cdot0072578$ - - log. $0\cdot3026031$

$\sqrt[3]{11}$ - - - $0\cdot3471309$

the greateſt root $= 4\cdot4641016$ - - $0\cdot6497340$

118. Again,

$\alpha = \frac{1}{3}A = \cdot3333333$ | $\beta = \frac{5}{9}B = \cdot0020406$

$\gamma = \frac{11}{15}C = \quad 330$ | $\delta = \frac{13}{21}D = \quad 7$

$+ \cdot3333663 \qquad - \cdot0020413$

$- \cdot0020413$

$\cdot3313250$

2

$\cdot6626500$ - - log. $\bar{1}\cdot8212842$

$\sqrt{3}$ - - - $0\cdot2385606$

$\sqrt[3]{121}$ - - - $0\cdot6942618$

the latter series $\pm$ $\cdot2320508$ - - $\bar{1}\cdot3655830$

$\frac{1}{2}$ the first $- 2\cdot2320508$

middle root $= - 2\cdot4641016$

least root $= - 2\cdot0000000$

119. But, by Art. 64, the 3 roots are -2 and $1 \pm \sqrt{12}$; hence

$$\frac{1+2\sqrt{3}}{2\sqrt[3]{11}} = 1 + \frac{2.2^3}{3.6.11^2} - \frac{2.5.8.2^4}{3.6.9.12.11^3} + \frac{2.5.8.11.14.2^6}{3.6.9.12.15.18.11^6} \text{ \&c.}$$

$$\frac{2-\sqrt{3}}{4}\sqrt[3]{121} = \frac{1}{3} - \frac{2.5.2^3}{3.6.9.11^3} + \frac{2.5.8.11.2^4}{3.6.9.12.15.11^4} - \text{ \&c.}$$

120. And in this manner the roots of cubic equations may always be found by these series; and then by comparing them with the roots of the same equations, as found by other methods, we shall obtain as many series as we please, whose sums will be given.

121. Hence

121. Hence alſo we may find the ſum of any general ſeries of either of theſe forms, namely,

$1 \mp \frac{2g^2}{3.6} - \frac{2.5.8g^4}{3.6.9.12} \mp \frac{2.5.8.11.14g^6}{3.6.9.12.15.18}$ &c. or

$\frac{1}{3} \pm \frac{2.5g^2}{3.6.9} + \frac{2.5.8.11g^4}{3.6.9.12.15} \pm \frac{2.5.8.11.14.17g^6}{3.6.9.12.15.18.21}$ &c. by comparing them with the roots of given cubic equations; whatever be the value of g, not greater than 1.

122. For, by Art. 68, $\sqrt[3]{b+c} + \sqrt[3]{b-c} = 2\sqrt[3]{b} \times : 1 - \frac{2c^3}{3.6b^3} - \frac{2.5.8c^6}{3.6.9.12b^6}$ &c. is = the greateſt root of the cubic equation $x^3 - 3\sqrt[3]{b^2-c^2}.x = 2b$. Now make $2\sqrt[3]{b} = 1$, and $\frac{c^2}{b^2} = g^2$; ſo ſhall the above become $\frac{1}{2}\sqrt[3]{1+g} + \frac{1}{2}\sqrt[3]{1-g} = 1 - \frac{2g^2}{3.6} - \frac{2.5.8g^4}{3.6.9.12}$ &c. = the greateſt root of the equation $x^3 - \frac{3}{4}\sqrt[3]{1-g^2}.x = \frac{1}{4}$. And when g^2 or $\frac{c^2}{b^2}$ is negative, theſe become

$\frac{1}{2}\sqrt[3]{1+g\sqrt{-1}} + \frac{1}{2}\sqrt[3]{1-g\sqrt{-1}} = 1 + \frac{2g^2}{3.6} - \frac{2.5.8g^4}{3.6.9.12} +$ &c. = the greateſt root of the equation $x^3 - \frac{3}{4}\sqrt[3]{1+g^2}.x = \frac{1}{4}$. So that in general the infinite ſeries

$1 \mp \frac{2g^2}{3.6} - \frac{2.5.8g^4}{3.6.9.12} \mp \frac{2.5.8.11.14g^6}{3.6.9.12.15.18}$ &c. is

$= \frac{1}{2}\sqrt[3]{1+g\sqrt{\pm 1}} + \frac{1}{2}\sqrt[3]{1-g\sqrt{\pm 1}}$ = the greateſt root of the equation $x^3 - \frac{3}{4}\sqrt[3]{1 \mp g^2}.x = \frac{1}{4}$. Where the upper and under ſigns reſpectively correſpond to each other.

123. Again,

123. Again,

$\sqrt[3]{c+b} - \sqrt[3]{c-b} = \frac{2b}{\sqrt[3]{c^2}} \times : \frac{1}{3} + \frac{2.5b^4}{3.6.9c^4} + \frac{2.5.8.11b^6}{3.6.9.12.15c^6}$ &c.

is the leaſt root of the equation $x^3 + 3\sqrt[3]{c^2-b^2} \cdot x = 2b$. Then, by taking $\frac{2b}{\sqrt[3]{c^2}} = 1$, and $\frac{b^2}{c^2} = g^2$, this becomes

$\frac{\sqrt[3]{1+g} - \sqrt[3]{1-g}}{2g} = \frac{1}{3} + \frac{2.5g^2}{3.6.9}$ &c. = the leaſt root of the equation $x^3 + \frac{3\sqrt[3]{1-g^2}}{4g^2}x = \frac{1}{4g^2}$. And when g^2 or c^2 is negative, this becomes

$\frac{\sqrt[3]{1+g\sqrt{-1}} - \sqrt[3]{1-g\sqrt{-1}}}{2g\sqrt{-1}} = \frac{1}{3} - \frac{2.5g^2}{3.6.9} +$ &c. = the leaſt root of the equation $x^3 - \frac{3\sqrt[3]{1+g^2}}{4g^2}x = \frac{-1}{4g^2}$. So that in general the infinite ſeries

$\frac{1}{3} \pm \frac{2.5g^2}{3.6.9} + \frac{2.5.8.11g^4}{3.6.9.12.15} \pm \frac{2.5.8.11.14.17g^6}{3.6.9.12.15.18.21}$ &c. is

$= \frac{\sqrt[3]{1+g\sqrt{\pm 1}} - \sqrt[3]{1-g\sqrt{\pm 1}}}{2g\sqrt{\pm 1}}$ = the leaſt root of the equation $x^3 \pm \frac{3\sqrt[3]{1\mp g^2}}{4g^2}x = \frac{\pm 1}{4g^2}$.

Of the Roots by another Claſs of Series.

124. But there are yet other ſeries, converging much faſter than thoſe in the foregoing claſs, by the help of which, and CARDAN's rule conjointly, may always be found

found the roots of thoſe equations in which that rule fails when it is applied ſingly, that is, in what is called the irreducible caſe, or that in which c^2 is negative. And thoſe ſeries are found by introducing another cubic equation having the ſame values of b and c^2 as the given equation, except that in the new equation the value of c^2 is poſitive, while in the given one it is negative. For when c^2 is poſitive, the new equation to which it belongs has only one real root, and that root is always found by CARDAN's rule; but the contrary takes place when c^2 is negative, the equation having then three real roots, although they are not always determinable by that rule, becauſe the radical quantities can ſeldom be extracted, on account of the ſquare root of the negative quantity which is contained in them.

125. Now the general expreſſion for the root by CARDAN's rule being $s + d = \sqrt[3]{b + \sqrt{\pm c^2}} + \sqrt[3]{b - \sqrt{\pm c^2}}$ or $\sqrt[3]{\sqrt{\pm c^2} + b} - \sqrt[3]{\sqrt{\pm c^2} - b}$, if the cubic roots of each of theſe be extracted by the binomial theorem, as at Art. 68, we ſhall obtain theſe 4 forms;

1. $\sqrt[3]{b + \sqrt{+c^2}} + \sqrt[3]{b - \sqrt{+c^2}} = 2\sqrt[3]{b} \times : 1 - \frac{2c^2}{3 \cdot 6b^2} - \&c.$

2. $\sqrt[3]{b + \sqrt{-c^2}} + \sqrt[3]{b - \sqrt{-c^2}} = 2\sqrt[3]{b} \times : 1 + \frac{2c^2}{3 \cdot 6b^2} - \&c.$

3. $\sqrt[3]{\sqrt{+c^2} + b} - \sqrt[3]{\sqrt{+c^2} - b} = \frac{2b}{\sqrt[3]{c^2}} \times : \frac{1}{3} + \frac{2 \cdot 5b^2}{3 \cdot 6 \cdot 9c^2} + \&c.$

4. $\sqrt[3]{\sqrt{-c^2} + b} - \sqrt[3]{\sqrt{-c^2} - b} = \frac{2b}{\sqrt[3]{c^2}} \times : -\frac{1}{3} + \frac{2 \cdot 5b^2}{3 \cdot 6 \cdot 9c^2} - \&c.$

126. Of

126. Of which the feries in the firſt and third denote the only real root of the equation when c^3 is poſitive, according as c is greater or leſs than b, which root call x; and the feries in the ſecond and fourth forms denote the greateſt and leaſt roots of the equation when c^3 is negative, which roots call R and r reſpectively. Then by adding and ſubtracting the firſt and ſecond, as alſo the third and fourth, there reſult theſe four equations;

$$\text{R} + \text{x} = 4\sqrt[3]{b} \times : 1 - \frac{2.5.8c^4}{3.6.9.12b^4} - \frac{2.5.8.11.14.17.20c^8}{3.6.9.12.15.18.21.24b^8} \text{ \&c.}$$

$$\text{R} - \text{x} = 4\sqrt[3]{b} \times : \frac{2c^3}{3.6b^3} + \frac{2.5.8.11.14c^6}{3.6.9.12.15.18b^6} + \text{\&c.}$$

$$\text{x} - r = \frac{4b}{\sqrt[3]{c^2}} \times : \frac{1}{3} + \frac{2.5.8.11b^4}{3.6.9.12.15c^4} + \text{\&c.}$$

$$\text{x} + r = \frac{4b}{\sqrt[3]{c^2}} \times : \frac{2.5b^3}{3.6.9c^3} + \frac{2.5.8.11.14.17b^6}{3.6.9.12.15.18.21c^6} + \text{\&c.}$$

127. And hence, by equal addition or ſubtraction, we find theſe two different expreſſions both for the greateſt and leaſt roots of a cubic equation in which c^3 or $b^3 + a^3$ is negative, namely,

$$\text{R} = -\text{x} + 4\sqrt[3]{b} \times : 1 - \frac{2.5.8c^4}{3.6.9.12b^4} - \frac{2.5.8.11.14.17.20c^8}{3.6.9.12.15.18.21.24b^8} \text{ \&c. or}$$

$$\text{R} = \text{x} + 4\sqrt[3]{b} \times : \frac{2c^3}{3.6b^3} + \frac{2.5.8.11.14c^6}{3.6.9.12.15.18b^6} + \text{\&c.}$$

$$r = \text{x} - \frac{4b}{\sqrt[3]{c^2}} \times : \frac{1}{3} + \frac{2.5.8.11b^4}{3.6.9.12.15c^4} + \frac{2.5.8.11.14.17.20.23b^8}{3.6.9.12.15.18.21.24.27c^8} \text{ \&c. or}$$

$$r = -\text{x} + \frac{4b}{\sqrt[3]{c^2}} \times : \frac{2.5b^3}{3.6.9c^3} + \frac{2.5.8.11.14.17b^6}{3.6.9.12.15.18.21c^6} + \text{\&c.}$$

where R is the greateſt, and r the leaſt root of the equa-

tion $x^3 - 3qx = 2b$ or $x^3 - 3\sqrt[3]{c^2 + b^2} \cdot x = 2b$, and x the only real root of the equation $x^3 + 3\sqrt[3]{c^2 - b^2} \cdot x = 2b$; in which, as well as in the above series, c^2 denotes a positive quantity.

128. And hence it can no longer be said that CARDAN'S rule is of no use in the solution of cubic equations that have three real roots; since they have here been reduced to the other case in which the equation has but one real root, which case is always resolvable by that rule. And the first hint of such reduction I received from FRANCIS MASERES, Esq. Cursitor Baron of the Exchequer, he having done me the favour to communicate to me the second of the above four forms for the greatest root, in a letter of the 17th of July 1779; the investigation of which formula, together with those of the other three, nearly as above, I had the honour of sending him in a letter of the 26th of the same month; and that learned gentleman has since communicated to the Royal Society his said formula, together with his own investigation of it, done in his usual very accurate manner. Since that time I have seen, in the *Memoires de l' Acad.* for the year 1743, four expressions similar to the above, given by Mr. NICOLE for the purpose of summing certain terms of a binomial raised to

any power, but unaccompanied with any appearance of the idea of thus reducing the one caſe of the cubic equation to the other.

129. It is hardly neceſſary to remark, that any general ſeries of each of the above four forms, is ſummed by means of the ſum or difference of the roots of theſe two equations $x^3 - 3\sqrt[3]{b^3 \pm c^3} \cdot x = 2b$, and that by ſubſtituting particular numbers for b and c, we may thus ſum as many ſeries of thoſe forms as we pleaſe.

130. Ex. 1. We may now illuſtrate theſe formulas by ſome examples. And firſt in the equation $x^3 - 15x = 4$. Here $2b = 4$, and $3\sqrt[3]{b^3 + c^3} = 15$, conſequently $b = 2$, and $c^3 = 5^3 - b^3 = 125 - 4 = 121 = 11^3$, and $x = \sqrt[3]{c+b} - \sqrt[3]{c-b} = \sqrt[3]{13} - \sqrt[3]{9} = .2712508$ the root of the equation $x^3 - 3\sqrt[3]{b^3 - c^3} \cdot x = 2b$ or $x^3 + 3\sqrt[3]{117} \cdot x = 4$. And as b is leſs than c, this equation belongs to the two ſeries in the latter caſe for finding the leaſt root. Hence, the terms of the two ſeries agreeing with the poſitive and negative terms of the ſeries in Art. 106, they will ſtand thus:

By

By the 1st series	By the 2d series
$A = .3333333$	$B = .0020406$
$C = .0000330$	$D = .0000007$
$.3333663$ - log. $\bar{1}.5229217$	$.0020413$ - - log. $\bar{3}.3099068$
$\frac{4^b}{\sqrt[3]{c^2}} = \sqrt[3]{\frac{512}{121}}$ - - - 0.2088282	$\frac{4^b}{\sqrt[3]{c^2}} = \sqrt[3]{\frac{512}{121}}$ - - - 0.2088282
series $= -.5392000$ $\bar{1}.7317499$	series $= +.0033016$ - $\bar{3}.5187350$
$X = +.2712508$	$X = -.2712508$
$r = -.2679492$ the least root	$r = -.2679492$ the same root.

Agreeing with the same root found in Ex. 4, Art. 106.

131. But the same root has been found to be $-2+\sqrt{3}$ in Art. 59, and hence we obtain the sums of these two particular series, thus,

$$\frac{\sqrt[3]{13}-\sqrt[3]{9}+2-\sqrt{3}}{8}\sqrt[3]{121} = \frac{1}{3} + \frac{2.5.8.11.2^4}{3.6.9.12.15.11^3} + \&c.$$

$$\frac{\sqrt[3]{13}-\sqrt[3]{9}-2+\sqrt{3}}{8}\sqrt[3]{121} = \frac{2.5.2^2}{3.6.9.11^2} + \frac{2.5.8.11.14.17.2^6}{3.6.9.12.15.18.21.11^5} \&c.$$

132. Also by taking the sum and difference of these two, we have

$$\frac{\sqrt[3]{13}-\sqrt[3]{9}}{4}\sqrt[3]{121} = \frac{1}{3} + \frac{2.5.2^2}{3.6.9.11^2} + \frac{2.5.8.11.2^6}{3.6.9.12.15.11^4} + \&c.$$

$$\frac{2-\sqrt{3}}{4}\sqrt[3]{121} = \frac{1}{3} - \frac{2.5.2^2}{3.6.9.11^2} + \frac{2.5.8.11.2^4}{3.6.9.12.15.11^3} - \&c.$$

And this last expression agrees with what was found in Art. 108.

133. Ex. 2. Again in the equation $x^3 - 9x = -10$, we have $2b = -10$, and $3\sqrt[3]{b^3 + c^3} = 9$; consequently $b = -5$, and $c^3 = 3^3 - b^3 = 27 - 25 = 2$, which being less than b^3 or 25, this equation belongs to the first class of series, or that for the greatest root. Now

$x = \sqrt[3]{b + c} + \sqrt[3]{b - c} = \sqrt[3]{-5 + \sqrt{2}} + \sqrt[3]{-5 - \sqrt{2}}$

$= -\sqrt[3]{5 - \sqrt{2}} - \sqrt[3]{5 + \sqrt{2}}$

$= -\sqrt[3]{3{\cdot}58578864} - \sqrt[3]{6{\cdot}41421356}$

$= -1{\cdot}530600 - 1{\cdot}858009 = -3{\cdot}388609$ = the root of the equation $x^3 - 3\sqrt[3]{b^2 - c^2}.x = 2b$, or $x^3 - 3\sqrt[3]{21}.x = -10$. And the terms of the two series are found as in Art. 110, namely $1 - \frac{2.5.8c^6}{3.6.9.12b^6} - \&c.$ $= A - C - E - \&c. = {\cdot}9997359$, and $\frac{2c^3}{3.6b^2} + \&c. = B + D + \&c.$ $= {\cdot}0089009$. Also $4\sqrt[3]{b} = 4\sqrt[3]{-5} = -4\sqrt[3]{5} = -\sqrt[3]{320}$. Then

By the 1st series		By the 2d series	
·9997359	- log. $\bar{1}$·9998854	·0089009	- log. $\bar{3}$·9494339
$-\sqrt[3]{320}$ - - -	0·8350500	$-\sqrt[3]{320}$ - -	0·8350500
series = - 6·838098	- 0·8349354	series = - ·060881	- $\bar{2}$·7844839
X = + 3·388609		X = - 3·388609	
- 3·449489 the greatest root		- 3·449490 the same root.	

And these values of the greatest root are nearly the same with that found in Art. 110.

134. But

134. But in Art. 61, the same root was found to be $-1-\sqrt{6}$, hence we obtain the sums of these first two particular series; and by the addition and subtraction of these two arise the other two following them, namely,

$$\frac{1+\sqrt{6}+\sqrt[3]{5+\sqrt{2}}+\sqrt[3]{5-\sqrt{2}}}{4\sqrt[3]{5}} = 1-\frac{2.5.8.2^3}{3.6.9.12.5^4}-\text{\&c.}$$

$$\frac{1+\sqrt{6}-\sqrt[3]{5+\sqrt{2}}-\sqrt[3]{5-\sqrt{2}}}{4\sqrt[3]{5}} = \frac{2.2}{3.6.5^2}+\frac{2.5.8.11.14.2^3}{3.6.9.12.15.18.5^6}+\text{\&c.}$$

$$\frac{1+\sqrt{6}}{2\sqrt[3]{5}} = 1+\frac{2.2}{3.6.5^2}-\frac{2.5.8.2^3}{3.6.9.12.5^4}+\frac{2.5.8.11.14.2^3}{3.6.9.12.15.18.5^6}-\text{\&c.}$$

$$\frac{\sqrt[3]{5+\sqrt{2}}+\sqrt[3]{5-\sqrt{2}}}{2\sqrt[3]{5}} = 1-\frac{2.2}{3.6.5^2}-\frac{2.5.8.2^3}{3.6.9.12.5^4}-\text{\&c.}$$

And the last but one of these equations agrees with one found in Art. 112.

135. Ex. 3. Also in the equation $x^3-12x=9$, we have $2b=9$, and $\sqrt[3]{b^2+c^3}=4$; consequently $b=\frac{9}{2}$, and $c^3=4^3-b^3=64-\frac{81}{4}=\frac{175}{4}$, which being greater than b^3 or $\frac{81}{4}$, this case belongs to the second class of series, or that of the least roots. Now here $x=\sqrt[3]{c+b}-\sqrt[3]{c-b}=\sqrt[3]{\frac{\sqrt{175}+9}{2}}-\sqrt[3]{\frac{\sqrt{175}-9}{2}}=\sqrt[3]{11\cdot114378}-\sqrt[3]{2\cdot114378}=2\cdot2316619-1\cdot2834950=\cdot9481669=$ the root of the equation $x^3-3\sqrt[3]{b^2-c^2}.x=2b$ or $x^3+3\sqrt[3]{\frac{47}{2}}.x=9$. And the terms of the two series being found as in Art. 113, namely, $A+C+E+$ &c. $=\cdot34051$, and $B+D+F+$ &c. $=\cdot03071$, also $\frac{4b}{\sqrt[3]{c^3}}$ being $=\frac{36}{\sqrt[3]{250}}$, we shall have

By

By the 1st series		By the latter series	
·34051 - log. $\bar{1}$·5321299		·03071 - log. $\bar{2}$·4872798	
$\frac{36}{\sqrt[3]{350}}$ - - -	0·7082798	$\frac{36}{\sqrt[3]{350}}$ - - -	0·7082798
series = - 1·739441 -	0·2404097	series = + ·1568771 -	$\bar{1}$·1955596
X = + 0·948167		X = - ·9481669	
- ·791274 the least root		- ·7912898 the same root	

Which nearly agree with the same root found in Art. 113.

136. But in Art. 62 the same root was found to be $\frac{3-\sqrt{21}}{2}$, hence then we shall have these first two following equations, and by means of their sum and difference we obtain the other two:

$$\frac{\sqrt[3]{20\sqrt{7}+36}-\sqrt[3]{20\sqrt{7}-36}+\sqrt{21}-3}{72}\sqrt[3]{350} = \frac{1}{3} + \frac{2.5.8.11.81^2}{3.6.9.12.15.175^2} \text{ &c.}$$

$$\frac{\sqrt[3]{20\sqrt{7}+36}-\sqrt[3]{20\sqrt{7}-36}-\sqrt{21}+3}{72}\sqrt[3]{350} = \frac{2.5.81}{3.6.9.175} + \text{ &c.}$$

$$\frac{\sqrt[3]{20\sqrt{7}+36}-\sqrt[3]{20\sqrt{7}-36}}{36}\sqrt[3]{350} = \frac{1}{3} + \frac{2.5.81}{3.6.9.175} + \text{ &c.}$$

$$\frac{\sqrt{21}-3}{36}\sqrt[3]{350} = \frac{1}{3} - \frac{2.5.81}{3.6.9.175} + \frac{2.5.8.11.81^2}{3.6.9.12.15.175^2} - \text{ &c.}$$

And the last of these agrees with one found in Art. 115.

137. Ex. 4. In the equation $x^3 - 15x = 22$, we have $2b = 22$, and $\sqrt[3]{b^2+c^2} = 5$; consequently $b = 11$, and $c^2 = 5^3 - b^2 = 125 - 121 = 4$, which being less than b^2 or 121, this belongs to the first class of series, or that for the greatest root.

Now

Now $x = \sqrt[3]{b+c} + \sqrt[3]{b-c} = \sqrt[3]{13} + \sqrt[3]{9} = 4.4314186 =$ the root of the equation $x^3 - 3\sqrt[3]{117}.x = 22$. And the terms of the two series being found as in Art. 117, we have the first $= A - C - \&c. = 1 - .0000450 = .9999550$, and the second $= B + D + \&c. = .0036731 + .0000008 = .0036739$. Also $4\sqrt[3]{b} = 4\sqrt[3]{11} = \sqrt[3]{704}$. Hence,

By the 1st series		By the 2d series	
.9999550 - log.	$\bar{1}.9999805$	.0036739 - log.	$\bar{3}.5651273$
$\sqrt[3]{704}$ - - -	0.9491909	$\sqrt[3]{704}$ - - -	0.9491909
series = + 8.8955200 -	0.9491714	series = + .0326827	$\bar{2}.5143182$
X = - 4.4314186		X = + 4.4314186	
+ 4.4641014 greatest root		+ 4.4641013 the same root	

Which nearly agree with the same root found in Art. 117.

138. But in Art. 64, the same root was found to be $1 + \sqrt{12} = 1 + 2\sqrt{3}$, hence we obtain these two first equations following, and their sum and difference give the other two:

$$\frac{1+\sqrt{12}+\sqrt[3]{13}+\sqrt[3]{9}}{4\sqrt[3]{11}} = 1 - \frac{2.5.8.2^4}{3.6.9.12.11^4} - \frac{2.5.8.11.14.17.20.2^8}{3.6.9.12.15.18.21.24.11^8} - \&c.$$

$$\frac{1+\sqrt{12}-\sqrt[3]{13}-\sqrt[3]{9}}{4\sqrt[3]{11}} = \frac{2.2^2}{3.6.11^2} + \frac{2.5.8.11.14.2^6}{3.6.9.12.15.18.11^6} + \&c.$$

$$\frac{\sqrt[3]{13}+\sqrt[3]{9}}{2\sqrt[3]{11}} = 1 - \frac{2.2^2}{3.6.11^2} - \frac{2.5.8.2^4}{3.6.9.12.11^4} - \frac{2.5.8.11.14.2^6}{3.6.9.12.15.18.11^6} - - \&c.$$

$$\frac{1+\sqrt{12}}{2\sqrt[3]{11}} = 1 + \frac{2.2^2}{3.6.11^2} - \frac{2.5.8.2^4}{3.6.9.12.11^4} + \frac{2.5.8.11.14.2^6}{3.6.9.12.15.18.11^6} - + \&c.$$

The last of which agrees with one found in Art. 119.

= And thus we may find the ſums of as many ſeries of theſe kinds as we pleaſe; as well as the ſum of any of the general ſeries, by means of the roots of given cubic equations. As to the ſummation of other forms of ſeries by means of the roots of equations of other orders, I ſhall perhaps treat of them on ſome future occaſion.

XXVI. *An Account of a moſt extraordinary Degree of Cold at* Glaſgow *in* January *laſt; together with ſome new Experiments and Obſervations on the comparative Temperature of Hoar-froſt and the Air near to it, made at the* Macfarlane *Obſervatory belonging to the College. In a Letter from* Patrick Wilſon, *M. A. to the Rev.* Nevil Maſkelyne, *D. D. F. R. S. and Aſtronomer Royal.*

Read April 20, 1780.

SIR, Glaſgow College, Feb. 27, 1780.

THE obſervations and experiments related in the ſequel were made here during the great froſt in the courſe of laſt month. I ſhall be extremely happy if they contribute any thing to your entertainment, being induced to ſend you the account in conſequence of your having on a former occaſion ſo politely invited me to a further correſpondence. If the paper ſhall appear to

you deſerving the notice of the Royal Society, you will pleaſe to conſider it as entirely at your own diſpoſal.

Moſt part of the time my father was confined to his room with bad health, and could not therefore gratify his curioſity by taking a part in the obſervations. I have now the pleaſure of acquainting you, that he is quite well again, and on this occaſion he begs to be remembered to you with beſt compliments.

On Tueſday, Jan. 11th, there was a ſlight froſt, and on the evening of that day we had a fall of ſnow to the depth of twelve inches. Next day the cold continued to increaſe, but ſo gradually, that at ſun-ſet FAHRENHEIT's thermometer pointed only to 22°. Upon returning home to the College from a viſit about midnight, I hung out a very accurate thermometer at a high North window, where it ſoon after pointed to 6°. At this time the air was very ſtill and ſerene, and the barometer ſtood at 30 inches.

Thurſday

Thurſday morning, Jan. 13, 1 o'clock, thermometer pointed to	gr. + 6
$1\frac{1}{4}$	+ 6
$2\frac{1}{2}$	+ 4
3	+ 6
4	+ 3
$4\frac{1}{2}$	+ 2
5	+ 2
$5\frac{1}{2}$	0

At 6 o'clock this morning I carried the thermometer over to the Obſervatory Park, and when it was there laid down upon the ſnow the mercury ſunk to gr. 13 below 0. Your friend Profeſſor ANDERSON ſoon after arriving, we repeated the obſervation with an excellent thermometer made by Mr. RAMSDEN, and it agreed exactly with the other.

At this time we thought it unneceſſary to ſtay abroad ſo long in the cold as to try the temperature of the air by hanging up the thermometers, eſpecially as we imagined that this had been done more readily, and as truly, by taking the degree from the ſurface of the ſnow which had been

expoſed to the open air during the night; but upon reflecting afterwards on the ſnow at the Obſervatory being ſo much below 0, the greateſt cold of the air at the College, and having on other occaſions found a difference of only 4° at moſt in air at theſe two ſtations, I was led into a ſuſpicion that the ſnow might perhaps have been ſo far cooled down by an evaporation at the ſurface. With a view to this opinion I projected the experiment with the bellows deſcribed below, by which I was not without expectations of producing a ſtill more remarkable fall of the thermometer when lying on the ſnow. Before dinner this day I met with Dr. IRVINE, to whom I communicated the above obſervations, and the intention of trying the evaporation; for that ſnow and ice did actually evaporate in ſome circumſtances ſeemed ſufficiently proved by an experiment made here by my father in 1768, which is related in the Philoſophical Tranſactions, vol. LXI. p. 326. Dr. IRVINE approved much of the propoſal, and moſt obligingly agreed to ſpend the night at the Obſervatory, that we might, by a regular courſe of obſervations, aſcertain the difference of temperature, and try whether evaporation was really the cauſe of it.

All the afternoon the cold was very intenſe, and at ſeven o'clock at night the thermometer at the high North window pointed to gr. 0. At eight Dr. IRVINE and I repaired to the Obſervatory, and made choice of a ſtation at a ſufficient diſtance from the houſe, and to the windward, as a light air was felt coming from the Eaſt. Here we laid down two thermometers on the ſnow with their balls half immerſed, and hung up other two freely expoſed to the air at two feet and a half from the ſurface.

In the following obſervations the interruption of the ſeries from $2\frac{1}{2}$ to $6\frac{1}{2}$ o'clock was owing to an accident having befallen one of the thermometers whilſt the other was employed in the trials, of which an account is ſubjoined.

Thurſday,

Thurſday evening, Jan. 13.

		Below 0.
8½ o'clock, therm. on the ſnow pointed to	gr. −12	Therm. in air to gr. 0
9	−14	− 2
10	−14	− 4
11	−17	− 6
11½	−18	− 6
½ o'clock Friday morning	−20	− 8
1	−23	− 7
1½	−22	− 8
2	−22	− 9
2½	−21	− 8
3		− 9
3½		−10
4		−12
4½		−12
5		−12
5½		−12
6		−14
6½	−22	−13
7	−22	−13
7½	−22	−13
8	−19	−10

EX-

EXPERIMENT I.

At half paſt one o'clock, when the thermometer pointed to − 22°, the ſnow contiguous to the ball was blown upon for two minutes by a pair of hand-bellows, held with the pipe nearly horizontal, and half a foot above the ſurface of the ſnow. The bellows had been lying out on the ſnow to cool from the time we firſt came over; and, in order to promote their cooling, they were now and then wrought in the open air. Care was alſo taken to ſtand to leeward of the thermometer, and to extend the bellows as far as poſſible from the body in the time of blowing. We were ſurpriſed to find, however, notwithſtanding all our precautions, that the thermometer at the end of the experiment had got up no leſs than 10°, for it now pointed only to − 12°. In this experiment the noſle of the bellows was held about ſix inches from the thermometer, but the blaſt, though moderate, frequently drifted away the ſnow from the ball.

EXPERIMENT II.

At half paſt two o'clock, a bread-baſket was filled with ſnow, taken up near the ground at + 14°. The contents being relatively ſo warm, the baſket was placed

to leeward of the common ſtation, and the thermometer laid on the ſurface of this ſnow.

At 3 o'clock in the morning, thermometer on the baſket pointed to . . .	−10°
3½	−15
4	−16½
4½	−18
5	−18
5½	−18
6	−18

EXPERIMENT III.

At four o'clock in the morning, when the thermometer in the baſket had got down to − 16°, a piece of thin fir plank about a foot ſquare was laid on the ſnow, upon which was placed a ſmall plate of tin which accidentally lay at hand. Upon this was laid one of the thermometers which had been hanging in the air.

At 5 o'clock, therm. on the plate pointed to	−16
5½	−16
6	−18

At ſeven this morning Profeſſor ANDERSON made us a viſit, and had the ſatisfaction of ſeeing the thermometers whilſt they indicated ſuch an extraordinary degree of cold.

cold. During the whole time not a cloud was perceivable, but there was a faint haze in the air when viewed towards the horizon. There was little or no tremor in the atmoſphere, which made the ſtars to ſhine with a full and ſteady light like that of the planets. Many of the citizens, who had thermometers hung out at their windows in different places of the town, found them pointing ſeveral degrees below 0 at nine o'clock in the morning. On the afternoon of this day, being Friday, Jan. 14th, the air became much warmer, and the barometer had now fallen four-tenths. Next day a thaw came on, and continued for ſome time.

As the above experiment with the bellows favoured ſo little the opinion, that the difference of temperature was cauſed by evaporation, I began to think, that it might be owing to a continued deſcent of cold air ſomewhere in the neighbourhood, which cooled the ſnow by ſweeping along its ſurface in ſo thin a ſheet as not to affect the air a little higher up. But as this ſolution of the phenomenon was ſo arbitrary, and unſupported by any facts, I wiſhed for another opportunity of making further experiments, and of enquiring into circumſtances ſtill more attentively. A good occaſion offered on Saturday, Jan. 22. The froſt, which before this time had again returned, became on this night very keen;

and a good deal of the former ſnow yet remaining on the ground, the following obſervations and experiments were made whilſt I was favoured with the company and aſſiſtance of Dr. ADAIR CRAWFORD, who paſſed the whole of this night at the Obſervatory.

On Sunday morning, Jan 23,

½ o'clock, therm. on the ſnow pointed to	+4	Therm. in air	+14
¾	+5		+14
1¼	+4		+11
1¾	+3		+11
2¼	+3		+11
2¾	+3		+11
3½	+1		+ 8
4	+1		+ 6
4½	0		+ 6
5	−1		+ 5
5½	−1		+ 6
6¼	−1		+ 6
7	0		+ 6
7½	−3		+ 5
7¾	−2		+ 5
8½	+1		+ 7

EX-

EXPERIMENT IV.

This night, inſtead of blowing on the ſnow, we fanned it by means of a ſheet of brown paper fitted to the end of a long ſlender ſtick. This apparatus was previouſly cooled by lying on the ſnow, and in fanning we took care to ſtand to leeward of the thermometer. The effect was, that the mercury roſe nearly to the ſame degree given by the thermometer in air at the ſame time.

EXPERIMENT V.

At three quarters paſt one o'clock, when the thermometer on the ſnow pointed to + 3°, it was ſcreened by two ſheets of brown paper ſet up on their edges, and ſo inclined againſt one another as to ſtand. The paper had been previouſly cooled by lying on the ſnow. At a quarter paſt two o'clock the thermometer thus ſheltered pointed to +9°. This experiment was afterwards repeated with the ſame event.

EXPERIMENT VI.

We next went up to the leads of the Eaſt wing of the Obſervatory. Here we hung a thermometer to the hook of a long pole, and raiſed it in the air about twenty-four feet from the ground, and at the ſame time inclined the pole over the balluſtrade, ſo as to put the inſtrument fully to windward of the houſe.

Upon ſuddenly lowering the pole, after half an hour, and examining the thermometer, the air at that elevation was found to be pretty conſtantly 4° warmer than at the ſtation below.

EXPERIMENT VII.

The reſult of this trial appeared more remarkable than any thing which had hitherto occurred. We lowered the pole till the thermometer was brought down within half a foot of the balluſtrade, but keeping it ſtill a few inches to windward of the building, and by this means it was found that the air here was never colder than +10°. Upon the balluſtrade there happened to be

ſeveral detached bodies which had attracted a very thick hoar-froſt. When the thermometer was taken off the hook of the pole, and laid on this hoar-froſt, there was always a remarkable fall of the mercury, not leſs than 6°. Both Dr. CRAWFORD and I were much ſtruck with this fact, and attended to it very carefully.

In tranſlating the inſtrument from the pole to the balluſtrade, it was commonly laid on ſome hoar-froſt three quarters of an inch deep, which had ſettled on a piece of thin board which had been for years expoſed to the weather. Some fragments of the hoar-froſt were alſo made to touch the upper part of the ball; which was done by puſhing them on with a long frozen ſtraw.

EXPERIMENT VIII.

When the thermometer taken from the pole as in laſt experiment was laid on pieces of ſtone, from which the hoar-froſt had been bruſhed away for ſome time before, the mercury ſunk but very little by ſuch a change of ſituation.

Next night, being that of Sunday Jan. 23. the thermometers were placed in their former ſtation below, when at 9 o'clock

9 o'clock at night, therm. on ſnow pointed on	+5	Therm. in air to	+9
9½	+5		+ 8
10	+6		+ 8
10½	+6		+10
11	+6		+ 9
11½	+5		+ 8
12	+5		+ 8
12½	+4		+ 7
1 o'clock Monday morning	+4		+ 8
1½	+4		+ 8

From theſe obſervations it appears, that the cold now was very moderate when compared to that of the 14th, and ſomewhat more moderate than that of the preceding night. Experiment 7th was again repeated with a ſimilar reſult, though the difference of temperature was not now ſo great.

This night I made another experiment with a view to the evaporation, not ſo liable to objections as thoſe of the bellows and the fan. This ſhall now be deſcribed.

EXPERIMENT IX.

When the thermometer in air at the lower ſtation had contracted a conſiderable film of frozen matter all over

over the ball, it was ſwung round at the end of a packthread, about a yard and a half long. Upon ſtopping the motion at the expiration of two minutes, and making the ſervant who waited approach quickly with a lighted candle, I found the mercury had got up 2°. In this experiment, which was repeated four times with the ſame reſult, care was always taken to keep the inſtrument to windward of our bodies, and of the lighted candle.

This night the thermometer, when put under the ſnow cloſe to the ground, pointed to +18°. Both now and on the preceding night there were no clouds, though the air, as on the 14th, was a little thick towards the horizon, and there was always a very perceptible and conſtant motion of it from Eaſt to Weſt. On this and the preceding night the Moon, a little paſt the full, ſhone very bright, and the mercury was ſtationary at 30 inches.

On Sunday, Feb. 13. we had a ſlight froſt accompanied with a thick fog. This occaſioned a vaſt ſettling of hoar-froſt upon the ſmall branches and twigs of trees, and upon all thin and detached bodies expoſed to the open air. The weather continued much in this ſtate on Monday, when, at ten o'clock forenoon, I repeated the ſeventh experiment on the balluſtrade, but now found no difference whatever in the temperature of the hoar-froſt and the air in its neighbourhood. The thermome-

ter in both caſes pointed to +22°. About 11 o'clock the ſun-ſhine broke through the fog, when the temperature of the hoar-froſt was as quickly affected as that of the air. The air on this day was quite ſtill, and equally cold both at the upper and under ſtations. It may alſo be here obſerved, that no ſnow lay on the ground. The ninth experiment was alſo repeated, but the thermometer was not now in the leaſt affected by ſwinging it round. Neither on this occaſion, nor when the experiment was firſt made, did any of the frozen matter appear to have parted from the ball.

If we conſider the exceſs of cold in the ſnow which Dr. IRVINE and I firſt obſerved as a phenomenon of the ſame kind with that deſcribed in the ſeventh experiment, and proceeding from the ſame cauſe, it is manifeſt that neither the one nor the other can be accounted for by any previous cold ſtate of the air, according to an hypotheſis alluded to in the beginning of this letter: for in the ſeventh experiment the air at the balluſtrade was never colder than +10°, and yet the hoar-froſt there was at the ſame time found in ſeveral inſtances as cold as +2°. That both theſe phenomena are of the ſame kind appears extremely probable from this conſideration, namely, that when the ſnow upon the fields was attentively examined, the ſurface was found quite covered

over

over with the ſame ſort of hoar-froſt which was attached to other bodies which had been long expoſed.

The two following experiments afford ſome grounds for believing that no kind of evaporation was going on at the time the remarkable exceſs of cold in the ſnow and hoar-froſt was obſerved.

EXPERIMENT X.

On Sunday morning, Jan. 23. before one o'clock, Dr. CRAWFORD and I repeated the experiment with the metal ſpeculum which was tried here in 1768. A large ſpare metal of a two-foot teleſcope was laid out to cool, after which a film of ice was imparted to its poliſhed ſurface by breathing on it four or five times. It was then expoſed as before, and in half an hour the whole film diſappeared in the way of evaporation. But when the experiment was again repeated, and a thicker film imparted, ſome of this, towards the middle of the ſpeculum, remained fixed, and would not go off after long expoſure. The ſpeculum was next warmed, and its poliſhed ſurface made quite clean, and then laid out for two hours and a half. Before the expiration of this time it began to draw frozen matter from the air, which ſettled all over the poliſhed ſurface in long parallel lines which

gradually multiplied, till at length it was moſtly covered with a thin film reſembling a ſpider's web.

The evaporation ſhewn in the firſt part of this experiment was probably owing to the ſpeculum not having been ſufficiently cooled when the film was firſt communicated to it from the lungs, and to its being further heated by that very operation. In the ſecond part of the experiment the evaporation ſeems to have ſtopped when the heat in the metal which favoured the proceſs was exhauſted; that is, when the ſpeculum had arrived at the temperature of the ambient air, for after that no heat could paſs from the metal in order to contribute to the evaporation. But from the laſt part of the experiment, the true diſpoſition of the air at that time, relative to bodies as cold or colder than itſelf, ſeems to be determined, namely, that of giving out or depoſiting hoar-froſt.

EXPERIMENT XI.

On Sunday night, January 23, ſeveral things were laid out at the Obſervatory, ſuch as ſheets of brown paper, pieces of boards, plates of metal, glaſſes of ſeveral kinds, &c. which all began to contract hoar-froſt ſeemingly as ſoon as each body had time to cool

down to the temperature of the air. The ſheets of brown paper being ſo thin acquired it ſooneſt, and when beheld in candle-light they became beautifully ſpangled over by innumerable reflections from the ſmall cryſtals of hoar-froſt which had parted from the air.

Evident ſymptoms of the ſame tendency of the air to depoſit occurred on all the former nights of obſerving, by which the tubes of the thermometers were ſo much ſtained, that it required ſome attention to keep that part which correſponded to the ſcale quite clear.

Theſe experiments indeed rather favour the opinion of the exceſs of cold at preſent treated of depending upon a principle the very reverſe of evaporation. But till opportunities offer in this or in a colder climate of making more experiments, it will be too early to ſay any thing deciſive concerning the nature or extent of a COOLING PROCESS which has ſo recently come under obſervation. All that can at preſent be affirmed is, that in certain circumſtances ſuch a proceſs goes on, and that it depends probably upon principles different from evaporation or chymical ſolution. At the ſame time ſome of the experiments ſhew that a free communication betwixt the hoar-froſt and external air, perhaps whilſt in motion, is neceſſary; but in what manner this promotes the REFRIGERATION doth not as yet appear.

Thoſe gentlemen whoſe opportunities have led them to know how far the philoſophy of heat hath, of late times, been extended by the great diſcoveries of Dr. BLACK, Dr. IRVINE, and Dr. CRAWFORD, will be moſt intereſted in the views which offer from the foregoing experiments. If at firſt ſight there appears any thing adverſe to certain general principles already eſtabliſhed, yet we may reſt ſatisfied that this is but in appearance only; as the perfect method of induction, purſued by theſe philoſophers, leaves no room to apprehend that any future diſcovery will militate againſt their concluſions. In the further proſecution of this ſubject, and in whatever way it may be cleared up, it is probable, that we ſhall meet with a fine inſtance of the congruity of nature in all her operations, and of the ſtability of thoſe general laws, which have been derived from a cautious obſervance of the rules of the experimental philoſophy.

I am, &c.

The following obſervations which relate to the diſpoſition of the air in giving out hoar-froſt may be here ſubjoined.

It would be going too far were we to conclude from the experiments related above, "that very cold air is

"never

"never diſpoſed to depoſit its contents except upon bo-
"dies as cold or colder than itſelf." And yet that this is frequently the caſe ſeems probable from a number of common appearances. We often find, after a night of froſt, the ſlates and other thinner parts about a houſe whitened with hoar-froſt, when the walls and more ſolid parts of the building remain quite free. In like manner the ſmaller branches and twigs of trees often acquire this frozen ornament, when the main branches and trunk remain naked for a long time; and, in general, any thin or detached body, capable of being eaſily cooled, attach hoar-froſt the ſooneſt.

In favour of this general poſition, the following remarkable caſe lately occurred, which at the time I ſhewed to Dr. REID, in conſequence of whoſe approbation I am induced at preſent to bring it into view.

Between the public library and the buildings of the new court there is a long rail compoſed of bars of caſt iron, but divided into two parts by two maſſy ſtone pillars which ſupport the iron gate-way that leads into the garden. The bars are about ſix feet high and an inch ſquare, and faſtened with lead into a ſtone parapet below in the uſual way. A few bars much larger are ſet in among the reſt at regular diſtances, in order to give the rail more ſtability. On Sunday morning, Feb. 13, when

we

we had the ſlight froſt accompanied with a fog, it was entertaining to obſerve how the hoar-froſt had ſettled during the night upon theſe bars. Very little was to be ſeen upon the flat ſides, but a great deal upon the angles, by which means from the top downward every bar was garniſhed with four fringes, which made the whole rail look very gay and ornamental. Running the eye along the foot of the bars near to the parapet it was obſerved, that the fringe of hoar-froſt upon the corners ſtopped ſhort about twelve inches from the bottom, and that ſo much of every bar was entirely free. Two bars next the houſe and two next the library were likewiſe perfectly clear of it from top to bottom. One bar next the pillar of the gate was quite free, and the ſecond had contracted but little. The ſame thing precifely may be ſaid of the two bars contiguous to the other pillar. And it was alſo obſerved, that the few thicker and ſtronger bars were leſs fringed at the corners, and were quite free much farther above the parapet than the others.

It is manifeſt, that during the night the air ſurrounding the bars muſt have been conſtantly endeavouring to make them as cold as itſelf, whilſt they, on the other hand, reſiſted this change by drawing heat from every neighbouring ſource which offered it, namely, from the parapet, from the pillars in the middle, and from the pillars

at

at both ends immediately adjoining to the library, and to the houſe in the new court: for theſe bodies from their great bulk muſt have been but very little cooled in the courſe of the night. Wherever the air ſeems to have got the better in this ſtruggle, as at the angles of the bars, which evidently muſt be the parts the ſooneſt cooled, there we find that the hoar-froſt was depoſited, but no where elſe.

Several other inſtances were found quite of the ſame kind with that of the rail. Among the reſt, a figure of an unicorn in ſtone, which ſtands within the college, had reſiſted the attacks of the air all to the tip of his horn, which accordingly was the only part diſtinguiſhed by a patch of hoar-froſt. Beſides this kind of hoar-froſt which joined itſelf to bodies by a regular arrangement, there was ſome of a different ſort found upon the uppermoſt ſurface of ſuch bodies as were fully expoſed to the open air. But this always lay ſcattered like very thin flakes of meal, or hair-powder, and was found to proceed from minute parts, moſtly columnar, previouſly formed in the air, falling down by their own gravity.

XXXVI. *Abſtract of a Regiſter of the Barometer, Thermometer, and Rain, at* Lyndon, *in* Rutland, 1779. *By* Thomas Barker, *Eſquire. Communicated by* Thomas White, *Eſquire, F. R. S.*

Read May 11, 1779.

		Barometer.			Thermometer.						Rain.
					In the Houſe.			Abroad.			
		Higheſt	Loweſt.	Mean.	High.	Low.	Mean	High.	Low.	Mean	
Jan.	Morn.	30,13	29,22	29,81	44½	34	38½	44	25	34	0,212
	Aftern.				45	35	39	49½	29	38	
Feb.	Morn.	30,06	29,26	29,77	49	41	46	44	29	40	2,239
	Aftern.				52	42½	47	55½	40	49	
Mar.	Morn.	30,13	29,18	29,73	53	42½	47	46	26½	38	0,131
	Aftern.				55	44	48	60	42	50½	
Apr.	Morn.	30,07	28,94	29,49	56½	45	51	52	32½	43½	1,888
	Aftern.				63½	48½	53	72½	47	55	
May	Morn.	29,84	29,14	29,44	64	45½	54	59	36	48½	1,266
	Aftern.				65½	46	55½	76	45½	59½	
June	Morn.	29,82	29,00	29,52	62	55	58½	60½	40½	53	2,416
	Aftern.				63½	56	60	74	59	63½	
July	Morn.	29,94	28,87	29,47	73½	60	65	66½	53	59	4,036
	Aftern.				75½	61	67	81½	60½	72	
Aug	Morn.	29,89	29,24	29,62	70½	61	64½	65	49	58	1,508
	Aftern.				75	62	67	81½	61	72	
Sept.	Morn.	29,71	29,05	29,42	68	55	61	60½	42½	53	1,227
	Aftern.				69	57	62½	73	55	65	
Oct.	Morn.	29,97	28,89	29,50	60	50½	54	55	34	45½	1,769
	Aftern.				59	52½	55½	64	47	56	
Nov.	Morn.	29,86	28,24	29,19	54	36	44½	50	24½	38	2,050
	Aftern.				54	36½	45	57½	31½	43½	
Dec.	Morn.	29,87	28,35	29,21	50½	30	40½	52	14½	35	3,136
	Aftern.				50	31	41	55	22½	38½	
											19,878

The end of winter and beginning of ſpring was warmer in 1779 than in 1778; the end of ſpring and beginning of ſummer 1778 was hotter than 1779; the end of ſummer and autumn 1779 hotter than in 1778; but the winter 1779 was much colder than that after 1778.

After the great ſtorm of January 1. came all the froſt there was this winter, which was not much; ſome broken froſts in the firſt nineteen days of January, after which there was ſcarce any at all. The weather was uncommonly mild, and more like ſpring than winter, and ſo dry that there was not much more than half an inch of rain in the firſt three months, and but an inch and a half in four months from Dec. 12. to Apr. 12. it was like that of the fine winters after 1742, 49, and 60, or that more remarkably dry, mild, and fine one after 1733, of which ſome account is given in the Ladies Diary for 1735; but many ſaid they never remembered ſo mild a winter: and what was remarkable, this winter, which ſeems to have been a mild one in all the North of Europe, was reckoned a ſevere one in the South of it.

February was ſo mild and fine that the wall-fruit flowered, and had better weather, and ſet much fuller, than the apples, which were two months later; the graſs grew alſo very conſiderably. The ſpring

ſeed-time was very fine, and every thing very forward, the vines put out in March, and the weather was in general fine, mild, and dry, though not without ſome cold N.E. winds, as is uſual in ſpring. The middle of April was quite hot, but the end of that month and beginning of May were cold, very ſhowery, and blaſted the apples, which were ſcarce this year; but that rain was very uſeful, for the graſs wanted it, and there was alſo a ſcarcity of water in many places; the reſt of May was hot and dry, or ſometimes hot ſun and cold winds.

It was a hot ſummer, and in general dry, yet with ſuch fits of rain as kept it from burning too much; but graſs was upon the whole rather ſcarce, and there was a great want of water in many places. The year was an uncommon one for fine weather, chiefly hot, and the fruits forwarder than uſual. Hay was not plentiful, but well got, and the bees were remarkably rich. A wet fit in wheat harveſt made ſome of it grow both cut and ſtanding, but in general it was well got, and the reſt of the harveſt fine, and all plentiful except peaſe; and grain cheaper than it has been for many years: wheat 26 or 28 ſhillings a quarter, and barley 15 or 16. The latter end of Auguſt was very hot, after which it grew gradually more moderate, but was ſtill dry and burning.

Some

Some ſhowers at the end of September and in October made the ground in good order for ſowing wheat, which came up very well, and was early. The autumn was mild and fine, and ſcarce any froſty mornings till near the middle of November, after which it was very winterly; either froſty, as it was the latter half of November, and from December 22. to February; or wet, as it was part of November, and the firſt three weeks of December. The froſts this winter were very ſevere, but never laſted long without ſome breaks.

Though the ſummer was very fine, it does not appear to have been healthy; there has been more illneſs than uſual ever ſince. In Auguſt (eſpecially in the low countries) colds were almoſt univerſal, and there were many ſore throats and fevers.

XXIX. *Journal of the Weather at* Senegambia, *during the Prevalence of a very fatal putrid Diſorder, with Remarks on that Country. By* J. P. Schotte, *M. D.; communicated by* Joſeph Banks, *Eſq. P. R. S.*

Read May 11, 1780.

SIR, London, Feb. 18, 1780.

HAVING kept a meteorological journal at the iſland St. Lewis, in the river Senegal, in Africa, during a time when the greateſt part of the garriſon, and a great number of the inhabitants on the iſland as well as on the continent, died of a putrid diſorder, I communicate the ſame to you, as I thought this fatal circumſtance a ſufficient reaſon to make it acceptable to the Society over which you preſide; hoping, that it may afford matter to determine the cauſe of it, and lead to find out remedies to prevent it in future in the like climates.

Previous to the journal, in order to illuſtrate it, I think it requiſite to make a few remarks on the ſituation of the iſland, the country about it, its ſeaſons, the manner and

time in which the diſorder appeared and ceaſed, and the thermometer I have uſed.

The iſland St. Lewis, otherwiſe called Senegal, is ſituated in 16° North latitude, and 16° Weſt longitude. It is ſeparated from the iſland of Soar on the Eaſt by the main river, which, on account of the ſmallneſs of the creek by which it is formed, is eſteemed a part of the continent. It has the Atlantic Ocean on the Weſt, from which it is ſeparated by a ſmall neck of land, or more properly ſand, called Barbary Point. This neck of land is in ſeveral places not above five or ſix hundred yards broad. A branch of the river runs between it and the iſland itſelf, communicating with the main river above and below the iſland. It is about a mile in length, ſeven hundred feet in breadth, and contains five or ſix thouſand black inhabitants. In the months of Auguſt, September, and October, it is uſually about two or three feet above the level of the river at high water; but there are years in which the whole iſland is overflowed; in the other months of the year it may be about five or ſix feet above its level in the higheſt places. The continent and iſlands near it are as low, and in many places much lower, being overflowed for the moſt part during the rainy months; the latter are formed by creeks communicating with the main river, and thickly beſet with mangroves.

mangroves. The water of the river is freſh during the rains, but very thick and troubled, the current being ſo rapid and ſtrong as to ſtop the flood-tide; but in the dry months the river water is ſalt, and no other water is to be had, but ſuch as is procured by digging a pit into the ſand more or leſs deep according to the height of the ground into which the water filtrates from all ſides, and gathers up to the level of the river. This water is brackiſh, but as no better is to be had thereabouts, the garriſon, as well as the inhabitants, make uſe of it, except when the river is freſh.

The year is commonly divided by the Europeans as well as the inhabitants into two ſeaſons; *viz.* the rainy and dry; by others called the ſickly and healthy ſeaſon. The rainy or ſickly one generally begins about the middle of July, and ends about the middle of October; during this time the wind is generally between the points of Eaſt and South, the quarter from which the tornados come. It has been obſerved, that this ſeaſon is more or leſs unwholeſome in proportion to the greater or leſſer quantity of rain that falls. A tornado is preceded by a diſagreeable cloſeneſs and weight in the air, (which ſeems to be much hotter than the thermometer ſhows it to be); and it is known to come on by the riſing of the clouds to the South-eaſt, which by joining grow darker,

darker, ſo as to make the horizon look quite black, accompanied with lightning and thunder at a diſtance. The breeze dies away by degrees as the tornado advances, and an entire calm ſucceeds; the air grows yet darker; animals and birds retire and ſhelter themſelves; every thing is ſilent, and the aſpect of the ſky, from whence the tornado approaches, is moſt dreadful. A violent ſtorm comes on all at once, which is ſo cold as to occaſion the thermometer to fall ſeven or eight degrees in a few minutes, and ſtrong enough to overſet negro huts and veſſels, or drive the latter from their anchors, and throw them on ſhore. The ſtorm abates, and heavy rain follows accompanied with much lightning and ſtrong claps of thunder. Sometimes tornados happen without rain, or at leaſt with very little, but then the ſtorm is more violent and laſts longer. It has been imagined by ſome, that this kind of ſtorm brings ſome peſtiferous quality with it, becauſe they had obſerved, that out of a number of people ſeveral fell ſick in one night after a tornado.

This I have in ſome degree experienced myſelf; for in the month of September 1776, feeling myſelf very well, and having dined as uſual, the ſtorm of a tornado ſuddenly tore down the window-ſhutters, and blew into the room where I was: about an hour after I had rigors,

and in the evening I had a high fever, which turned out to be a very ſevere bilious one; but notwithſtanding this, it has, in my opinion, no ſuch ill quality, and the above phenomenon may be attributed to the change it produces on the air, and of conſequence on the body; it may therefore be conſidered as the occaſional cauſe of a diſorder to which the body was pre-diſpoſed long before.

The dampneſs of the atmoſphere during this ſeaſon is ſo great that it is more or leſs perceptible in every thing. Leather, wearing apparel, and books, grow mouldy. Poliſhed metals grow ruſty. Sea ſalt, ſugar, and other ſaline ſubſtances, which were perfectly dry before, melt; and the meat of cattle killed in the evening is ſpoiled the next morning, ſo as not to be fit for uſe.

Calms are very frequent and diſagreeable on account of the muſquetoes and other inſects, which then quit their retreats from among the mangroves and marſhes, and ſpread over the face of the country.

The dry or healthy ſeaſon begins commonly about the middle of October and laſts to the middle of July. It is called dry, becauſe then it hardly ever rains, or at leaſt but very ſeldom; and healthy, in oppoſition to the ſickly one: for though pleuriſies and peripneumonies will happen in the months of December and January, and

fluxes

fluxes in the months of April, May, and June, few people die, which, when compared with the numbers that die in the other ſeaſon, juſtifies the denomination. When the rains ceaſe, the wind ſhifts its quarter, and is for the moſt part Eaſt or North-eaſt in the morning; but as the Sun riſes on the horizon, the wind changes more and more towards the North, till about noon, ſooner or later, it gets to the Weſt of North, which is called ſea-breeze, and is very refreſhing, though it happens ſometimes, that as the Sun falls again on the horizon, the wind will ſhift again towards the Eaſt, and continue there all night. This wind blows ſometimes very ſtrong, and is always exceſſively hot, drying up the lakes and pools, which had been formed by the heavy rains and the overflowing of the river, and producing in ſuch as partake of ſea water, a fine ſea ſalt in large cryſtals, not unlike foſſil ſalt. In the months of February, March, April, May, and June, the wind blows almoſt conſtantly from between North and Weſt, called ſea-breeze, except now and then a day or two it will be Eaſt, which when it happens in April makes it exceſſively hot, the Sun being then in and about the zenith of Senegal, heating the vaſt plains of ſand over which this wind is to paſs before its arrival there, which, reverberating the received heat, may contribute to increaſe it; for I have obſerved, that the

ſame month in the river Gambia was not hotter than any other wind, owing in all appearance to the difference of the ſoil of the country, which is not ſandy like that of Senegal. I think it is the duſt of the ſand raiſed by this wind which makes the atmoſphere look hazy. I myſelf ſaw in the year 1775, in the month of April, in a morning preceded by an Eaſterly wind, ſuch a duſt imitating a fog in the air, that one could not ſee above twenty yards.

The weather grew calm, and about eleven o'clock in the forenoon the atmoſphere grew clear by depoſiting a browniſh impalpable duſt, which covered every thing near a line in thickneſs. The ſame thing I obſerved at ſea from on board of a veſſel in the month of March 1775, at the diſtance of about five or ſix leagues from the land near the latitude of Senegal. The wind having blown Eaſt in the night, I found in the morning the ſails, ſhrouds, and deck, covered with an impalpable duſt. The deſcription given by the learned Dr. LIND [a] of the Harmattans of the Coaſt of Guinea, ſeems to agree with the Eaſt wind at Senegal in almoſt every reſpect, except that the damp vapour in the former is not perceptible in this, for it dries every thing that will admit of it. Water poured on the floor of a

(a) Eſſay on the moſt effectual Means of preſerving the Health of Seamen.

room

room for the purpoſe of cooling the air, is dried up in an inſtant, and there is ſome effect on the thermometer placed in ſuch a room. Salt, ſugar, and the like ſubſtances, which are half melted by the damp air during the rainy ſeaſon, dry again in a few days into hard lumps. Such houſhold furniture as is made of wood, though it has been ever ſo well ſeaſoned, ſhrinks and grows looſe where joined, or ſplits and cracks where glued. It dries and parches the ſkin of the white people as well as the blacks, and makes it ſometimes as rough as any clear froſty weather in Europe would. The ſky is commonly clear and without clouds; but the atmoſphere is hazy, which, in my opinion, as I have already obſerved, is occaſioned by the duſt, perhaps in conjunction with vapours ariſing from the ſurface of the earth and waters. Theſe vapours, though not to be ſeen in the open air, I have perceived by their ſhadow upon white walls, ariſing from pools which were cloſe to them; but the air being ſo dry they are abſorbed by it, and no more perceptible as vapour. That the evaporation muſt be very great when this wind blows, the method the blacks have of cooling water will evince. They fill tanned leather bags with it, and hang them up in the Sun; the water oozes more or leſs through the leather, ſo as to keep the outward ſurface of it wet, which, by its quick and con-

tinued evaporation, occaſions the water within the bag to grow conſiderably cool.

This wind is in general not reckoned unwholeſome, either by the inhabitants or Europeans, though it feels very diſagreeable, and by depriving the body of its thinner fluids may be looked upon as the immediate cauſe of ſome diſeaſes, and the pre-diſpoſing one to others. When it ſets in ſooner or later in the month of October, it is conſidered by the inhabitants as producing a ceſſation of the ſickly weather, and the beginning of healthier. In the months of December and January, when the Sun is at its greateſt diſtance, it makes the weather feel very cold in the nights and the mornings.

The putrid diſorder, which proved ſo fatal to the garriſon and the inhabitants of Senegal, made its appearance in the beginning of Auguſt. The preceding month of July had been remarkably healthy; though the weather was very hot and ſultry, there were only three ſoldiers in the hoſpital for ſlight venereal diſorders; but we learnt by ſome black meſſengers, who came from Goree, that there was a fever raging there, which had carried off numbers of the French garriſon and inhabitants of the iſland, and we thought ourſelves very happy not to partake of their fate. On the ſecond of Auguſt one of the ſoldiers, who was in the hoſpital for a gonorrhœa, being

cured, was discharged. The fourth of August he was again reported to me as very sick in the barracks. I went to see and found him in a high fever with the worst symptoms. I ordered him to be carried to the hospital, where he died the third day, with all the symptoms of the greatest putridity. The orderly man of the hospital was seized on the sixth of August with the same disease, and died the ninth. One of the venereal patients, who remained still in the hospital, was taken with the same fever, and died a few days after. Some of the soldiers of the fort, having access to the hospital to visit their sick comrades, took the contagion, and spread it through the whole garrison. I am apt to believe, that the disorder was brought to Senegal by the black messengers from Goree; for I understood that one of them had died soon after his arrival in Senegal, and it may be, that the soldier who died first of it got the infection from them; for it is probable, that being discharged the hospital on the second of August, and having leave to take a walk on the island on the third, he had been in company with some of these black messengers, or in the huts where they resorted, for the sake of hearing some news from Goree, where he was acquainted. It may perhaps be observed, that the soldier taking the contagion on the third of August, it could not make so rapid a progress as

to

to manifeſt itſelf the next morning in the higheſt degree; but this I intend to ſupport by the following caſes. One of the ſurgeon's mates dreſſed a bliſter on the back of a ſoldier, ill of the diſorder, with a digeſtive ſoftened with oil of turpentine: having done, he came into the ſurgery, and looked quite pale, telling me, That the ſoldier's back had ſmelled ſo putrid and offenſive, that it had made him quite faint and ſick at the ſtomach. He took ſome tincture of bark and bitters, and went home, when a fever, with a train of the worſt ſymptoms, made its appearance in the evening, and he died the third day. Another gentleman, who was ſent for by the ſaid ſurgeon's mate in the morning of the ſecond day of his illneſs, and requeſted to draw up a will for him, arrived while I was preſent. He ſpoke with the patient for a few minutes, and then took me aſide, ſaying, That there was a certain ſmell about the room, which made him faint and ſick at the ſtomach, and that he ſhould be obliged to retire; he did, but in the evening was ſeized with the fever and all its bad ſymptoms, went through ſeveral of its ſtages, but recovered. A black boy, who had been waiting on the ſaid ſurgeon's mate during his illneſs, was taken with the ſame diſorder, and died of it in a few days. I could produce ſeveral other caſes to ſtrengthen what I have advanced concerning the quick

appearance of the diſorder itſelf after the contagion had taken place, but I think the three related ones ſufficient.

The ceſſation of this contagious diſeaſe may be dated from about the middle of September. Governor CLARKE, who died the 18th of this month, concluded the dreadful ſcene. He had avoided the communication with all ſick people, but did not heſitate in admitting my company. I was the only one who dined with him for ſeveral weeks; and as I was continually among the ſick in the hoſpital and on the iſland (of the former of which I gave him a return every morning) I might probably have conveyed the infection to him in my cloathing, though I was not affected myſelf. A few people died in the months of October, November, and December; ſome of relapſes of the ſame fever, and others of ſevere fluxes and abſceſſes in the liver, in which the diſorder had terminated. It is remarkable, that a fleet of merchant-men, under convoy of a ſloop of war, which left Senegal on the fourth of Auguſt, and ſailed for England, had, by what I could learn, been entirely free from this diſorder; neither did it reach as far as the river Gambia, for the garriſon at Fort James in that river enjoyed a pretty good ſtate of health during all this time, and loſt only two men, who died of fluxes.

The thermometer I have uſed is FAHRENHEIT'S made by WILLCOX and COYSGARNE. It has been compared, ſince my arrival in England with one made by RAMSDEN, and found to be about three-fourths of a degree lower. It was placed in the fort at Senegal, in a room two ſtories high with a blank cieling, and above that a bevil roof covered with ſlate: this room was not expoſed to the Sun but at its riſing and ſetting, it being ſheltered from it by other buildings, when high on the horizon. I am ſorry that I had not begun to keep the journal of heat and weather two or three months ſooner, for the ſatisfaction of the curious in natural philoſophy; but as it required a degree of leiſure which I had not, being obliged to attend to my duty, and as I obſerved nothing extraordinary either in the heat or weather, and imagined that journals of this kind, and of this country, might have been publiſhed in Europe long before now, I neglected it, but was tempted to keep it when the fatal diſeaſe made its appearance.

It has been obſerved by Dr. LIND [b], that in that country a great change in the weather has little or no effect on the barometer. I have remarked the ſame at Fort James Gambia in the year 1776, for I found from the fourth of February to the laſt of April, that the alte-

(*b*) Eſſay on Diſeaſes incidental to Europeans in hot Climates.

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ration the weather produced on the barometer was ſo little as hardly to be perceptible. The equality of the weather during this time (which is part of the dry ſeaſon in which the ſky is always clear and without clouds, though the different winds produce ſenſible changes in the atmoſphere) may perhaps account for it; but Governor CLARKE, who had a barometer placed in one of his rooms in the fort at Senegal, told me, That the greateſt changes in the weather during the rainy ſeaſon had ſo little effect on that inſtrument that it was hardly worth notice.

I beg you would do me the honour to preſent theſe remarks, with the annexed journal, to the Royal Society, if you find them to contain any thing worthy the notice of that illuſtrious body.

I have the honour to be, &c.

1778	Hour.	Therm.	Wind.	Weather.
Aug. 10	11	82	NNW	Clear.
	2	83	NNW	A tornado coming on, which lasted an hour.
	4	81	NNW	Lightning to the Southward.
	8	81	NNW	Lightning and thunder to the Southward.
11	6	80	N	Clear.
	9	81	N	Clear.
	4	83	W	Lghtning and thunder with rain.
	8	81	W	Clear with lightning.
12	6	76	NNE	Thund. lightn. and rain during the whole night.
	11	80	S	Clear.
	3	82	N	Clear.
	8	82	——	Calm and clear.
13	7	80	SE	Small rain.
	11	82	SSW	Cloudy with Sun-shine; close and moist.
	4	83	W	Clear.
	9	81	W	
14	7	80	W	Cloudy with small rain.
	11	82	W	Cloudy.
	4	83	W	Cloudy.
	9	82	W	Cloudy.
15	7	80	NNW	Cloudy.
	5	83	W	Cloudy.
	9	81	W	Cloudy.
16	7	80	SW	Clear.
	11	82	NW	Clear.
	3	83	NW	Clear.
	9	82	N	Clear.
17	7	79	S	Cloudy.
	10	80	W	Cloudy.
18	7	80	S	Cloudy.
	12	82	NE	Thunder and rain.
	2	78	W	Thick and misty.
	8	79	WSW	Hazy.
19	7	78	ESE	Clear.
	11	81	S	Clear.
	4	84	NW	Cloudy with Sunshine.
	9	82	——	Calm and clear.
20	7	79	S	Cloudy.
	4	84	W	Cloudy and thunder.
	9	82	W	Cloudy and rain.

1778	Hour.	Therm.	Wind.	Weather.
Aug. 21	7	79	SSE	A great deal of rain.
	1	78	SW	Cloudy and hazy.
	4	78	SW	Cloudy and hazy.
	7	80	S	Cloudy and ſhowers of rain.
22	6	78	SW	A great deal of rain.
23	7	78	——	Calm and clear, a great deal of rain the night before.
	2	81	SW	Cloudy.
	9	80	NW	Clear, lightning to the Eaſtward.
24	7	79	N	Cloudy.
	12	82	NNW	Clear.
	2	82	——	Thunder, a tornado coming on from the Eaſt.
	9	81	——	Calm and cloudy, a tornado with rain three hours before.
25	7	78	——	Calm, a heavy tornado with rain laſt night.
	1	81	N	Cloudy.
	4	81½	N	Cloudy.
	10	81	N	Clear.
26	7	80	N	Clear.
	11	80	E	Clear, a tornado with rain two hours before.
	1	81	——	Calm and clear.
	11	82	N	Clear.
27	7	78	E	Miſty, a great deal of rain the night before.
	11	78	SE	Cloudy, now and then rain.
	3	80	SE	Cloudy.
28	7	78	SW	Miſty and drizzling rain.
	2	82	SW	Cloudy.
	6	81	SW	Cloudy.
	9	80	W	Cloudy.
29	7	80	W	Cloudy.
	1	81	NW	Cloudy, blowing hard.
30	7	80	NW	Cloudy, the air very thick.
	10	82	SE	Thick and hazy.
	2	83	WSW	Hazy.
	8	82	W	Hazy.
31	7	78	NE	Thick and rain; a tornado laſt night, with a deal of rain, thunder, and lightning; before the tornado came on it was exceſſive cloſe.
	3	83	E	Cloudy.
	9	83	——	Calm, cloudy, and very cloſe; lightning all round the horizon.
Sept. 1	7	80	W	Cloudy.
	3	83	WSW	Clear.
	9	82	W	Clear and fine.

1778	Hour.	Therm.	Winds.	Weather.
Sept. 2	7	81	SW	Close and cloudy.
	1	83	W	Hazy.
	5	84	NW	Thunder towards the South-east.
	9	84	NW	Lightning towards the South-east.
3	7	81	SE	Close and cloudy; last night calm and very close.
	11	84	E	Clear.
	5	85	SW	Cloudy.
	9	82	W	Cloudy.
4	7	80	W	Cloudy.
	11	80	W	Showers of rain with thunder.
	9	81	NW	Cloudy.
5	7	80	NW	Cloudy.
	12	83	NW	Cloudy.
	9	82	NW	Cloudy.
6	7	81	——	Calm and cloudy.
	2	84	W	Cloudy.
	4	84	SE	A tornado with rain.
	9	82	——	Calm and hazy.
7	7	81	SW	Cloudy.
	1	82	SW	Cloudy.
	5	84	——	Calm and close, a tornado coming on from the SE.
	9	80	SW	The tornado ceased, cloudy and damp.
8	6	78	S	Cloudy.
	12	83	S	Hazy and cloudy.
	9	82	NW	Hazy and cloudy.
9	7	82	SE	Cloudy and close.
	11	83	SW	Cloudy.
	6	84	W	Hazy and close.
	9	83	W	Cloudy.
10	7	82	SE	Cloudy.
	3	83	W	Cloudy.
	5	85	NW	Clear.
11	7	82	NW	Cloudy.
	5	83	N	Cloudy.
	9	82	N	Cloudy.
12	6	80	N	Cloudy.
	12	82	N	Cloudy.
	5	83	NW	Cloudy.
	9	82	NW	Cloudy.
13	6	80	N	Cloudy.
	3	83	N	Cloudy.
	5	83	N	Cloudy.
	9	81	NW	Cloudy.

1778	Hour.	Therm.	Winds.	Weather.
Sept. 14	7	81	SE	Cloudy and thick, laſt night very cloſe.
	12	83	SW	Cloudy and hazy.
	2	83	SW	A tornado coming on from the Eaſt.
	9	79	——	Calm, from three o'clock till now heavy rain.
15	7	79	SE	
	11	82	S	Cloudy and rain now and then.
	5	82	W	Cloudy and rain now and then.
	9	82	W	Cloſe and cloudy, lightning towards the Eaſt.
16	6	81	E	Cloudy, the night very cloſe.
	10	76	N	Two hours before a tornado from the Eaſt, with a deal of wind and cold rain.
	2	82	S	Cloudy.
	9	81	——	Calm, cloudy, and cloſe.
17	7	81	NNW	Clear, laſt night exceſſive cloſe, and millions of muſquetos.
	12	83	NW	Clear.
	5	84	NW	Clear.
	9	82	NW	Clear.
18				
19	7	82	——	Calm, the night before very cloſe.
	10	84	——	Calm, a tornado coming on from the Eaſtward.
	1	81	E	Clear, two hours ago a tornado with but little rain.
	5	84	NW	Clear.
	8	84	——	Calm and clear, the air filled with muſquetos.
20	7	81	N	Clear, the night before quite calm.
	1	84	WNW	Clear.
	9	84	W	Lightning to the Eaſtward.
21	7	81	E	Cloudy, laſt night a tornado with a great deal of wind, but not much rain.
	2	84	E	Cloudy.
	5	85	E	Cloudy.
	9	84	E	
22	8	79	E	Cloudy.
	1	85	——	Calm and clear.
	5	85	W	A tornado from the Eaſt.
23	11	84	NW	Clear.
	3	84	NW	Clear.
	9	83	NW	Clear.
24	7	82	S	Cloudy.
	10	84	SW	Hazy.
	3	87	——	Calm and clear.
	9	85	NNW	Clear.

1778	Hour.	Therm.	Winds.	Weather.
Sept. 25	7	82	E	Hazy and thick; at three o'clock this morning a heavy tornado, with a great deal of rain, thunder, and lightning.
	11	84	E	Hazy and thick.
	2	87	SE	Hazy.
	5	88	SE	Hazy.
	9	84	NW	Clear.
26	5	—	——	A tornado from the E. with a great deal of rain.
	8	79	E	Clear.
	4	84	——	Calm.
	8	84	——	Calm, plenty of musquetos.
27	9	83	N	Clear.
	2	85	N	Hazy.
	4	84	N	Hazy.
	8	84	N	Hazy.
28	6	83	——	Calm and hazy.
	1	85	SW	Hazy.
	5	86	——	Calm and hazy.
	8	84	W	Hazy.
29	7	82	W	Hazy.
	1	85	NW	Hazy.
	9	84	N	Clear.
30	7	82	N	Clear.
	1	84	NNW	Clear.
	5	85	NNW	Clear.
	9	84	NNW	Clear.
Oct. 1	8	81	N	Clear.
	1	84	N	Clear.
	9	83	N	Clear.
2	6	79	N	Clear.
	11	83	NNW	Clear.
	9	83	——	Calm and hazy.
3	8	80	E	Cloudy and hazy.
	12	80	E	Hazy.
	9	81	E	Hazy.
4	6	76	SE	Cloudy.
	1	81	SE	Cloudy, a few drops of rain now and then.
	9	81	——	Calm and cloudy.
5	6	79	——	Calm and clear.
	1	82	W	Calm and clear.
	9	82	W	Cloudy.

1778	Hour.	Therm.	Winds.	Weather.
Oct. 6	6	79	WSW	Cloudy, an hour before a tornado with rain and thunder.
	11	77	SE	Cloudy, 3 hours before a tornado with cold rain.
	2	77	W	Cloudy.
	9	79	N	Cloudy.
7	6	76	N	Clear.
	1	80	N	Clear.
	5	81	N	Clear.
	9	81	N	Clear.
8	7	80	N	Clear.
	10	82	N	Clear.
	5	84	N	Clear.
	9	83	N	Clear.
9	7	81	N	Clear.
	11	85	N	Clear.
	12	87	NNE	Clear.
	9	83	N	Clear.
10	7	80	W	Cloudy.
	11	82	W	Cloudy.
	1	83	SW	Cloudy.
	9	82	N	Cloudy.
11	6	80	——	Calm and cloudy.
	11	82	N	Clear.
	2	84	N	Clear.
	9	83	N	Clear.
12	6	80	NNE	Cloudy.
	12	82	NNW	Cloudy.
	9	82	N	Clear.
13	6	79	N	Clear.
	2	82	N	Clear.
	9	82	N	Clear.
14	6	79	N	Clear.
	10	80	N	Clear.
	4	81	N	Clear.
15	6	77	N	Clear.
	2	81	N	Clear.
	9	80	NNW	Clear.
16	6	78	N	Hazy.
	12	79	N	Clear.
	5	82	W	Clear.
	9	81	W	Clear.

1778	Hour.	Therm.	Winds.	Weather.
Oct. 17	6	79	W	Hazy, a heavy dew laſt night.
	12	82	——	Calm and hazy.
	9	81	W	Hazy, a dew.
18	6	79	W	Clear.
	12	82	W	Clear.
	6	83	W	Clear.
	9	83	W	Clear.
19	6	78	NNW	Clear.
	2	82	WNW	Clear.
	9	80	NW	Clear.
20	7	78	N	Clear, a heavy dew laſt night.
	2	82	NNW	Clear.
	9	80	N	Clear.
21	6	78	N	Clear.
	2	82	NNW	Clear.
	9	80	NW	——
22	7	79	NW	——
	1	81	N	Heavy dew laſt night.
	8	80	N	Clear.
23	6	76	N	Clear.
	11	82	NE	——
	9	81	W	Dew.
24	6	78	NW	——
	2	86	NE	Duſty.
	3	88	E	Duſty.
	4	90	NE	Duſty.
	5	92	E	Duſty.
	8	87	E	——
25	7	81	N	Clear.
	11	86	E	Duſty.
	3	92	E	Duſty.
	4	92	E	Duſty.
	9	83	N	——
26	8	77	NE	——
	4	85	N	Clear.
	9	81	N	Clear.
27	6	79	N	Clear.
	2	87	N	Clear.
	9	84	N	Clear.
28	6	79	E	——

1778	Hour.	Therm.	Winds.	Weather.
Oct. 29	6	78	N	Clear.
	11	84	NE	Dusty.
	1	90	NE	Dusty.
	2	92	NE	Dusty.
	9	83	NNW	——
30	6	78	N	Clear.
	12	86	NE	Dusty.
	9	83	NW	——
31	8	81	W	Hazy.
	11	82	W	Hazy.
	4	84	NW	Hazy.
	9	85	——	Calm.
Nov. 1	6	80	NNE	——
	1	84	NNW	Cloudy.
	5	83	N	Thunder and rain.
	9	83	N	Cloudy.
2	7	80	N	Hazy.
	1	90	NE	Dusty.
	4	91	ENE	Dusty.
	9	87	NNW	——
3	7	80	NNE	Dusty.
	12	90	NNE	Dusty.
	4	92	NNE	Dusty.
	9	85	NNW	——
4	7	80	——	Calm.
	11	82	NNW	Clear.
	5	83	NNW	Clear.
	9	82	NNW	Clear.
5	7	78	NNW	Clear.
	11	85	N by E	Clear.
	1	80	NNW	Clear.
	9	81	NNW	——
6	6	77	NW	Clear.
	3	82	NNW	Clear.
	9	80	NW	Clear.
7	6	78	NNW	Clear.
	11	79	NNW	Clear.
	9	80	NW	Clear.
8	7	79	SW	Thick and cloudy, dew last night.
	11	84	W	Hazy.
	9	80	W	Hazy.

1778	Hour.	Therm.	Winds	Weather.
Nov. 9	7	80	——	Calm.
	1	89	NE	Dusty.
	4	91	NE	Dusty.
	9	84	N	——
10	7	78	NNE	——
	11	85	NE	Dusty.
	4	91	NE	Dusty
	9	83	NW	——
11	6	76	NE	——
	2	91	E	Dusty.
	5	89	N	——
	9	82	E	——
12	7	76	——	Calm. Note, that from this date forward the sky and atmosphere have been always clear, when the wind blew from between North and West; but when it blew from between North and East, the atmosphere was more or less hazy and the sky clear, except on those days where there are particular remarks.
	2	89	E	
	3	92	E	
	9	83	NNE	
13	7	78	E	
	2	90	E	
	9	84	NNE	
14	6	76	ENE	
	2	90	NNE	
	9	83	NNE	
15	7	75	NE	
	4	87	NNE	
	9	82	NNE	
16	6	75	NNE	
	1	86	N by E	
	3	90	NNE	
	9	81	N by E	
17	7	75	NNW	
	11	81	N by E	
	4	82	NNW	
	9	80	NNW	
18	6	75	NNW	
	1	79	NNW	
	2	82	NNW	
19	6	74	NNW	
	12	79	NNW	
	2	79	NW	
	9	76	NNW	
20	7	74	NNW	
	11	76	NNW	
	9	78	NW	

1778	Hour.	Therm.	Winds.	Weather.
Nov. 21	7	73	NNW	
	1	85	NNE	
	3	88	NNE	
	9	79	NNW	
22	6	73	NNE	
	1	84	E	
	2	84	NW	
	9	78	NNW	
23	6	73	NNE	
	1	84	E	
	2	84	NW	
	9	78	NNW	
24	6	70	E	
	4	81	NE	
	5	79	NNE	
	9	78	NNE	
25	6	69	ENE	
	12	74	E	
	5	76	E	
	9	74	NE	
26	6	63	E	
	4	75	E	
	9	72	E	
27	6	64	E	
	12	74	E	
	6	74	NNE	
	9	72	NNE	
28	6	64	E	
	1	75	E	
	9	74	E	
29	6	64	E	
	11	72	E	
	4	77	N by E	
	9	72	NNE	
30	6	64	E	
	1	75	E	
	9	73	NNW	
Dec. 1	6	64	E	
	11	72	E	
	2	80	E	
	9	73	N	

1778	Hour.	Therm.	Winds	Weather
Dec. 2	6	64	E	
	1	79	E	
	4	77	NNW	
	9	73	NNW	
3	0	64	E	
	11	76	E	
	1	79	E	
	9	75	NNW	
4	6	65	NE	
	1	78	E	
	9	74	NNW	
5	6	67	NE	
	2	77	E	
	5	76	NNW	
6	6	68	NE	
	1	74	NNW	
	4	73	NNW	
7	6	68	NNE	
8	7	70	NE	
	1	76	NNE	
	9	73	N	
9	7	70	NNE	
	12	72	NNE	
	2	76	NNE	
	9	72	NE	
10	6	66	E	
	11	76	E	
	1	80	NE	
11	7	68	E	
	1	79	E	
	2	80	NNW	
	9	74	NNW	
12	7	65	NE	
	11	70	E	
	4	80	E	
	9	73	N	
13	7	66	NNE	
	11	76	E	
	3	80	E	
	9	74	NNW	

1778	Hour.	Therm.	Winds.	Weather.
Dec. 14	7	67	E	
	11	76	E	
	3	83	E	
	9	76	N	
15	7	71	E	
	1	78	E	
	5	86	E	
	19	79	NNE	
16	7	71	E	
	1	81	E	
	2	84	E	
	9	75	NNW	
17	7	69	NE	
	2	75	NE	
	4	78	N	
	19	72	N	
18	7	64	NE	
	2	84	E	
	4	79	N	
	9	73	N	
19	6	65	NE	
	1	80	E	
	9	75	NE	
20	6	62	NE	
	2	80	E	
	9	75	NNE	
21	6	64	NE	
	1	73	E	
	2	75	E	
	9	71	NNW	
22	6	64	NE	
	9	71	NNW	
23	6	64	E	
	12	78	E	
	2	81	NNE	
	9	73	NNW	
24	7	66	E	
	2	78	E	
	5	77	NNE	
	9	72	N	

1778	Hour.	Therm.	Winds.	Weather.
Dec. 25	7	63	NE	
	1	73	E	
	2	74	E	
	9	70	N	
26	7	58	NE	
	11	68	E	
	3	72	NNW	
	9	67	N	
27	7	60	NE	
	2	74	N	
	9	68	N	
28	7	62	NE	
	12	73	E	
	2	72	NNW	
	9	70	NNW	
29	6	70	E	Foggy and thick, drizzling rain.
	7	70	N	Small rain.
	11	72	E	
	1	75	E	
	9	72	N	A few drops of rain.
30	6	67	N	Thick and foggy, with a few drops of rain.
	12	71	NNW	Thick and foggy.
	4	73	N	Foggy.
	9	70	N	
1779				
Jan. 1	7	64	NE	
	1	76	E	
	4	80	NNW	
	9	72	NNW	
2	7	63	E	
	11	68	NNW	
	3	71	NNW	
	9	69	NNW	
3	6	64	NNW	
	11	70	NNE	
	3	70	NNW	
	9	69	NNW	
4	6	63	N	
	12	73	E	
	9	70	NNW	
5	6	60	N	
	9	69	NNW	

1779	Hour.	Therm.	Winds.	Weather.
Jan. 6	6	63	N	
	11	70	NNW	
	4	72	NNW	
	9	70	NNW	
7	6	62	N	Foggy.
	2	71	NNW	
	9	70	NNW	
8	7	67	NE	Cloudy and thick, lightning and rain last night.
	4	70	NNW	
	9	69	NNW	
9	7	67	N	Foggy.
	4	70	N	
	9	70	N	
10	7	67	N	
11	7	69	N	
	1	70	NNW	
	4	71	NNW	
	9	69	NNW	
12	7	67	N	
	2	70	NNW	
	4	70	NNW	
	9	69	NNW	
13	7	68	N	Dew last night.
	2	70	NNW	
	4	71	NNW	
	9	69	NNW	
14	8	68	N	Dew last night.
	1	69	NNW	
	4	69	NNW	
	9	68	NW	
15	9	68	NNW	Dew last night.
	11	69	NNW	
	4	70	NNW	
	9	69	NNW	
16	7	67	N	Heavy dew last night.
	11	68	NNW	
	4	69	NNW	
	9	69	NNW	
17	7	67	N	
	1	68	NNW	
	4	69	NNW	
	9	68	NNW	

1779	Hour	Therm.	Winds	Weather.
Jan. 18	7	66	NE	
	12	70	NE	
	4	73	NE	
	9	71	NE	
19	7	63	NE	
	11	68	NE	
	4	72	NE	
	9	69	N	
20	7	59	NE	
	11	70	NE	
	4	75	NE	
	9	69	N	
21	7	60	N by E	
	1	73	NE	
	4	75	NE	
	9	67	NE	
22	6	58	NE	
	12	70	NE	
	4	75	NE	
	9	67	NE	
23	6	58	NE	
	12	69	NE	
	2	71	NE	
	9	67	NE	
24	6	59	NNE	
25	6	55	N by E	
	12	67	NE	
	4	70	NE	
	9	65	NE	
26	6	56	NE	
28	6	55	NE	
	11	66	NE	

XXIX. *Aſtronomical Obſervations relating to the Mountains of the Moon.* *By Mr.* Herſchel *of* Bath. *Communicated by Dr.* Watſon, *Jun. of* Bath, *F. R. S.*

Read May 11, 1780.

AT the time when the teleſcope was firſt invented this noble inſtrument was immediately applied to aſtronomical obſervations with the moſt ſurpriſing ſucceſs. Several very eminent perſons have given us an account of their diſcoveries; and, notwithſtanding the imperfect ſtate of teleſcopes in thoſe times, we ſtill owe a great deal of our knowledge of the heavenly bodies to the obſervations that were made by thoſe firſt teleſcopic obſervers, who made amends for the deficiencies of their inſtruments by their uncommon diligence and attention.

It may, perhaps, be eſteemed to be a mere matter of curioſity to ſearch after the height of the lunar mountains. I grant that there are more neceſſary and more uſeful objects of inquiry in the ſcience of aſtronomy; but when we conſider that the knowledge of the conſtruction of the Moon leads us inſenſibly to ſeveral conſequences,

ſequences, which might not appear at firſt; ſuch as the great probability, not to ſay almoſt abſolute certainty, of her being inhabited, we ſhall ſoon agree, that theſe reſearches are far from being trifling.

My reaſon for repeating obſervations that have been made by very good aſtronomers was not that I doubted either their veracity or diligence. The names of GALILEO, HEVELIUS, KIRCHER, and ſeveral more, will always deſerve to be mentioned with particular reſpect for the eminent ſervices they have rendered to aſtronomy; but as we know that their inſtruments were far from being arrived to that degree of perfection we have now obtained, I thought it by no means improper or uſeleſs to repeat their obſervations on the lunar mountains, and to extend them to other parts of the Moon's viſible hemiſphere, and thereby to eſtabliſh this theory on the firmeſt evidence of a ſurvey taken by a very excellent inſtrument.

The method uſed by HEVELIUS and others to find the height of a mountain in the Moon is this. Let a ray of light SLM (fig. 1.) proceeding from the Sun, paſs by the Moon at L, and touch the top of a mountain at M: then the ſpace between L and M will appear dark, and the top of the mountain will be ſeen to ſtand at ſome diſtance from the illuminated part of the Moon's diſk. With a

good micrometer let the diſtance LM be taken by obſervation (*a*). Draw LC perpendicular to LM; draw alſo MC from the top of the lunar mountain to the center of the Moon: then in the triangle MLC, rectangled at L, we have given the ſide LC, which is the Moon's radius, and the ſide LM taken by obſervation. Therefore, by trigonometry, we can find the hypothenuſe MC (*b*), from which, ſubtracting the part *p*C or radius, there remains the perpendicular height of the mountain M*p*. I have followed the ſame method, as being the leaſt liable to error.

GALILEO takes the diſtance of the top of a lunar mountain from the line that divides the illuminated part of the diſk from that which is in the ſhade to be equal to a 20th part of the Moon's diameter; but HEVELIUS affirms, that it is only the 26th part of the ſame.

When we calculate from thence the height of ſuch a mountain it will be found, in Engliſh meaſure, according to GALILEO, almoſt $5\frac{1}{2}$ miles; and, according to HEVELIUS, ſomething more than $3\frac{1}{4}$ miles, admitting the Moon's diameter to be 2180 miles.

(*a*) I do not recollect that HEVELIUS mentions in what manner he took the diſtance LM; but I am apt to believe it was by a micrometer.

(*b*) $\sqrt{LC^2 + LM^2} = MC$.

He ſays, in his Selenography, p. 266. "Vera diſtan-
"tia illuſtratarum cuſpidum, à confinio luminis et um-
"bræ, præſertim tempore quadraturæ, invenitur, unâ
"vigeſimâ ſextâ parte, totius Lunæ dimetientis con-
"ſtare; quando nimirum ſunt remotiſſimæ: quemad-
"modum hoc ex phaſi trigeſimâ ſecundâ, monteque
"Apennino; ex phaſi trigeſima prima, monteque Di-
"dyme; et trigeſimâ phaſi, monteque Tauro et Anti-
"tauro, manifeſtiſſimè demonſtratur." Having after-
wards mentioned that GALILEO makes the diſtance LM to
be the 20th part of the Moon's diameter, HEVELIUS pro-
ceeds, "Quamobrem, cum diſtantiæ a nobis deſignatæ,
"paululum ſint minores, idcirco et montes aliquantu-
"lum depreſſiores inveniuntur, quàm GALILÆUS æſti-
"mavit: neque non tamen illi terrenis noſtris montibus,
"quoad altitudinem, non ſolum æquiparari poſſunt meri-
"tiſſimò; ſed et multo certe ſunt excelſiores, quàm
"noſtri omnium maximi; prout confeſtim, ex adjecto
"diagrammate patebit." He gives us then his calcula-
tion according to German and Italian meaſure; and hav-
ing found, in the manner above mentioned, the hypo-
thenuſe MC, he adds: "Semidiameter Lunæ erit 1976
"octav. part. Si igitur hæc à totâ hypothenuſa aufere-
"tur, reſtabunt adhuc ſex, hoc eſt ſex octavæ unius mil-
"liaris, vel tres quartæ unius milliaris Germanici, ſive
"tria

" tria milliaria Italica: quæ eſt vera, et genuina altitudo " iſtius montis." As a German mile in the time of HEVELIUS was a very uncertain meaſure, we may ſuppoſe that he meant geographical miles, 15 of which make a degree of latitude. The obſervations of HEVELIUS have always been held in great eſteem; and this is moſt probably the reaſon why later aſtronomers have not repeated them. M. DE LA LANDE, who is one of our moſt eminent modern aſtronomers, agrees to the ſentiments above cited.

In his *Abrégé d'Aſtronomie*, p. 485. he ſays, " Je ter- " minerai ce qui concerne la ſelenographie, en diſant un " mot de la hauteur des montagnes de la lune, qui étoient " quelquefois éclairées, quoiqu' éloignées de la ligne de " lumière de la treizième partie du rayon de la Lune; de " la on peut conclure que ces montagnes ont de hauteur " la 338^{ieme} partie du rayon Lunaire, ou une lieue de " France. He then gives us a particular calculation, and the reſult is: " Avec ces données on trouve la hauteur de " 2643 toiſes, c'eſt à dire, plus d'une lieue commune."

He alſo mentions the opinion of GALILEO, and adds: " Mais on doit préférer à cet égard les obſervations " d'HEVELIUS, qui ont été plus répétées, plus détaillées " et plus exactes."

Mr.

Mr. FERGUSON ſays (Aſtronomy explained, § 252.) "Some of her mountains, by comparing their height "with her diameter, are found to be three times higher "than the higheſt hills on our earth."

KEILL, in his Aſtronomical Lectures, has calculated the height of St. Katherine's hill, according to the obſervations of RICCIOLUS, and finds it nine miles.

Before I report my own obſervations, it will be neceſſary to explain by what method I have found the height of a lunar mountain from obſervations that were made when the Moon was not in her quadrature; for the method laid down by HEVELIUS will only do in that one particular caſe: in all other poſitions the projection of the hills muſt appear much ſhorter than it really is. Let SLM, or *slm* (fig. 2.) be a line drawn from the Sun to the mountain, touching the Moon at L or *l*, and the mountain at M or *m*. Then, to an obſerver at E or *e* the lines LM, *lm*, will not appear of the ſame length, though the mountains ſhould be of an equal height; for LM will be projected into *on*, and *lm* into ON. But theſe are the quantities that are taken by the micrometer when we obſerve a mountain to project from the line of illumination. From the obſerved quantity *on*, when the Moon is not in her quadrature, to find LM we have the following analogy. The triangles *o*OL, *r*ML, are ſimilar; therefore,

therefore, LO : LO :: L*r* : LM, or $\frac{LO \times on}{Lo}$ = LM; but LO is the radius of the Moon, and L*r*, or *on*, is the observed distance of the mountain's projection; and L*o* is the sine of the angle ROL = *o*LS, which we may take to be the distance of the Sun from the Moon without any material error, and which therefore we may find at any given time from an ephemeris.

I will now give an account of my own observations relating to the mountains in the Moon; but, perhaps, it may not be amiss to mention the instrument they were made with, and a few of the circumstances, that it may appear how far their accuracy may be depended upon.

The telescope I used was a Newtonian reflector of six feet eight inches focal length, to which a micrometer was adapted consisting of two parallel hairs, one of which was moveable by means of a fine screw. The value of the parts shewn by the index was determined by a trigonometrical observation of a known object at a known distance, and was verified by several trials. The power I always used, except when another is mentioned, was 222 times, also determined by experiment, which I have often found to differ somewhat from theory, on account of some little errors in the *data*, hardly to be avoided. The moon having sufficient light, I used no more

more aperture of the object ſpeculum than four inches; and, I believe, that for diſtinctneſs of viſion this inſtrument is perhaps equal to any that was ever made.

OBSERVATIONS.

November 30, 1779, ſix o'clock in the morning, a rock, ſituated near what HEVELIUS calls *Lacus niger major*, was meaſured to project 41″,56. To reduce this quantity into miles, put R for the ſemi-diameter of the Moon in ſeconds, as given by the Nautical Almanac at the time of obſervation, and Q for the obſerved quantity, alſo in ſeconds and centeſimals; then it will be in general R : 1090 :: Q : $\frac{1090\,Q}{R} = on$, in miles. Thus it is found that 41″,56 is 46,79 miles. This diſtance of the Sun and Moon at the ſame time was, by the Nautical Almanac, about 93° 57′½. The ſine of which to the radius 1 is .9985, &c. and $\frac{on}{Lo}$ in this caſe, is LM = 46,85 miles. Then, by HEVELIUS's method the perpendicular height of the rock is found to be about one mile.

The ſame morning, a great many rocks, ſituated about the middle of the diſk, projected from 25″,93 to 26″,56. This gives *on* about 29,3 miles, and theſe rocks are all leſs than half a mile high.

January

January 13, 1780, 7 o'clock, I examined the mountains in the Moon; but there was not one of them that was fairly placed on level ground, which is a condition very neceſſary for an exact meaſurement of the projection. If there ſhould be a declivity on the Moon before the mountains, or a tract of hills placed ſo as to caſt a ſhadow on that part before them which would otherwiſe be illuminated, it is plain that the projection would appear too large; and, on the contrary, ſhould there be a riſing ground before them, it would appear too little.

As far as I was able to judge of the direction of the line of illumination, the higheſt hill projected 25″31, or 30,36 miles: from thence we find, as before that the perpendicular height is (,42 mile) leſs than half a mile.

January 14, 11 o'clock, I took the projection of the higheſt mountain which was ſituated at the Weſtern edge. It meaſured 24″,68, or about 27 miles; and the perpendicular height comes out leſs than half a mile. There was not one mountain in the edge of the diſk ſo high as this.

January 17, 7 o'clock, a very high mountain projected no leſs than 40″,625. Its ſituation is in the South-eaſt quadrant. The Moon's ſemi-diameter, at the time of

obſervation, by the Nautical Almanac, was 16′ 2″,6; therefore, $\frac{10900}{R}$ = 45,98 miles = *on*.

	°	′	″
Sun's longitude at 7h. . .	9 27	39	0
Moon's longitude at 7 . .	2 2	46	52
Their neareſt diſtance, . .	4 5	7	52

or about 125° 8′; the ſine of which is .8104: thence we find LM 56,73 miles; and the perpendicular height of the mountain is 1m. 47, or leſs than a mile and a half.

January 22, 8h. 20′. the higheſt mountain, ſituated near Snell or Petavius, projected 11″,437, which is 12′,34; and LM comes out to be 35,3 mile: threfore the perpendicular height is ,57 mile.

Another, juſt behind Mare Criſium, meaſured only 7″, therefore is leſs than half a mile high.

January 25, 7h. 30′. in the morning, a mountain near Ariſtoteles meaſured 18″,59 which gives 20,6 miles; and LM is found 28,53 miles; the perpendicular height is therefore only ,37 mile.

Other mountains about Mare Nectaris meaſured about 23″,5; but they had hills before them, and their ſituation was not ſo proper for my purpoſe. However, it is evident they were of no conſiderable height.

January 28, 6 o'clock in the morning, the higheſt mountain in the diſk meaſured 30″,937; the Moon's ſemi-diameter at that time 15′ 40″; and *on* therefore equal 31,37 miles: but as the Moon is within four hours of her quadrature we may be aſſured that this mountain is leſs than half a mile high.

February 19, Mons Sinopium projected 5″,781; therefore *on* = 6,26 miles, and the quantity LM 56,54 miles; and conſequently the height of this mountain, which it ſeems proves to be a very high one, is not much leſs than a mile and a half. However, my journal obſerves, that the meaſure was very full; therefore the mountain in all probability does not exceed a mile and a quarter. Moreover, I think that obſervations made ſo near the full or new Moon are leſs to be depended upon, becauſe a ſmall error in meaſuring will produce a great one in the height of a mountain.

From theſe obſervations I believe it is evident, that the height of the lunar mountains in general is greatly over-rated; and that, when we have excepted a few, the generality do not exceed half a mile in their perpendicular elevation. It is not ſo eaſy to find any certain mountain exactly in the ſame ſituation it has been meaſured in before; therefore ſome little difference muſt be expected in theſe meaſures. Hitherto I have not had an

opportunity of particularly obſerving the three mountains mentioned by HEVELIUS; nor that which RICCIOLUS found to project a ſixteenth part of the Moon's diameter. If KEILL had calculated the height of this laſt mentioned hill according to the theorem I have given, he would have found (ſuppoſing the obſervation to have been made, as he ſays, on the fourth day after new Moon) that its perpendicular could not well be leſs than between eleven and twelve miles.

I ſhall not fail to take the firſt opportunity of obſerving theſe four, and every other mountain of any eminence; and if other perſons, who are furniſhed with good teleſcopes and micrometers, would take the quantity of the projection of the lunar mountains, I make no doubt, but that we ſhould be nearly as well acquainted with their heights as we are with the elevation of our own. One caution I would beg leave to mention to thoſe who may uſe the excellent $3\frac{1}{2}$ feet refractors of Mr. DOLLOND. The admirable quantity of light, which on moſt occaſions is ſo deſirable, will probably give the meaſure of the projection ſomewhat larger than the true, if not guarded againſt by proper limitations placed before the object glaſs. I have taken no notice of any allowance to be made for the refraction a ray of light muſt ſuffer in paſſing through the atmoſphere of the Moon,

when

when it illuminates the top of the mountain, whereby its apparent height will be leſſened, as we are too little acquainted with that atmoſphere to take it into conſideration. It is alſo to be obſerved, that this would equally affect the concluſions of HEVELIUS, and therefore the difference in our inferences would ſtill remain the ſame.

Bath, February 28, 1780.

Continuation of the ſame obſervations.

March 11, 1780, 7h. Promontorium Archerufia projected 17″,187. It is very properly ſituated for meaſuring. By a proper deduction from the Moon's ſemidiameter, as given by the Nautical Almanac, at the time of obſervation, we find the quantity *on* = 26.1 miles, and LM 22,6 miles; from which it appears, that the perpendicular height of this mountain is a little leſs than a quarter of a mile.

Antitaurus, the mountain meaſured by HEVELIUS was badly ſituated, becauſe Mount Moſchus and its neighbouring hills caſt a deep ſhadow, which may be miſtaken for the natural convexity of the Moon. A good, full, but juſt meaſure, 25″,105; in miles 29,27: therefore,

LM

LM 31,7 miles, and the perpendicular height not quite half a mile.

7h. 45′. I was deſirous of being very exact in this meaſure, therefore I repeated it. I took two different obſervations. A narrow meaſure 21″,562; quite full enough 24″,062. Theſe meaſures give the perpendicular height leſs than half a mile.

8h. I meaſured Lipulus, 19″063. It is alſo badly ſituated, though rather better than Antitaurus. I found that the projection increaſed, therefore concluded that this was not the higheſt part of the mountain, and waited ſome time when I meaſured it again.

9h. Lipulus now projected 28″,75.

10h. It meaſured 28″,75: this gives $on = 33,64$ miles. Diſtance of Sun and Moon about 63° 23′: therefore, LM 37,54 miles. From hence we find the perpendicular height ,64 mile, or very near two-thirds of a mile.

March 12, 1780, 7h. One of the Apennine mountains, between Lacus Traſimenus and Pontus Euxinus projected 44″,062. This gives us $on = 51,11$ miles; and LM = 52,9 miles: therefore the perpendicular height of theſe mountains, which I know to be very high, comes out to be $1\frac{1}{4}$ mile.

Mons

Mons Armenia (near Taurus) projected 31″406 = 36,43 miles; LM = 38 miles nearly, and the height two-thirds of a mile.

Mons Leucopetra 34″479 or 40 miles; LM = 41,4 miles, and the perpendicular height three quarters of a mile.

There was a very fine ſhade of a high rock near it, which ſhewed the direction of the illuminating ray, and thereby aſſiſted me in meaſuring to a great exactneſs; but the mountain itſelf is not very favourably ſituated.

March 16, 10h. 30′. Mons Lacer projected 45″,625; but I am almoſt certain that there are two very conſiderable cavities or places where the ground deſcends below the level of the convexity, juſt before theſe mountains, ſo that theſe meaſures muſt of courſe be a good deal too large: but ſuppoſing them to be juſt, it follows, that *on* is 50,193 miles, LM = 64 miles, and the perpendicular height above $1\frac{1}{4}$ miles.

Another of the ſame mountains ſituated on the borders of S. Sirbonis meaſured 41″,875. This ridge of mountains is the ſame of which I meaſured one on January the 17th, which was then found to be 1,47 miles high.

The

The following additional Memoranda of the Manner in which Mr. HERSCHEL *made his Obſervations are taken from a Letter of his to the rev. Dr.* MASKELYNE, *Aſtronomer Royal.*

IN the ſecond figure of my obſervations, the points L, S, E, *r*, are all ſuppoſed to be in one plane; and as the illuminating ray SL is alſo in this plane, it follows, that the line L*n* (= *o n*) will always be perpendicular to the right line which joins the cuſps of the Moon (*c*); and the truth of the theorem there delivered depends upon this circumſtance.

For this reaſon I have taken care in all my obſervations to meaſure the line, which in fig. 3. (taken from your letter to Dr. WATSON) is marked *o n*, parallel to the line CD, or perpendicular to AB, and not the line *r n*, perpendicular to the elliptical curve A*r*OB.

The manner of taking it is eaſy enough: however, I have occaſionally uſed three different methods, and will de-

(*c*) It is here ſuppoſed, that rays from the Sun S, and the eye of the obſerver E, to any part of the Moon L, may be taken for parallel; and therefore, that different planes, made by ſeveral ſections of the Moon, according as the point L is taken North or South of the diameter of the Moon, which is at rectangles to the line joining the cuſps, may alſo be taken to be parallel to that diameter.

ſcribe

ſcribe them all, which I ſhould have done in the paper delivered by Dr. WATSON, had I not feared to be too particular.

The firſt method I uſed was to ſet the immoveable hair *bb* (fig. 4.) of my micrometer parallel to a line AB, joining the cuſps of the Moon; then, by opening the moveable parallel hair till it included the projection *on*, intended to be taken, I marked that down as the meaſure of *on*. As this method required ſome attention (that part of the ellipſis of illumination AVB which is the vertex v of the leſſer axis may ſerve as a direction) and took up ſome time, on account of the ſmall field of view of my teleſcope, I uſed occaſionally theſe two following ways.

When there was any remarkable figure on the diſk of the Moon near the line of illumination, I put on a compound eye-piece whoſe magnified field of view is full 40', and power about 90 times, ſo that it takes in the greateſt part of the whole Moon; by this means I was enabled to view the projection intended for meaſuring at the ſame time with the reſt of the Moon, and to fix upon ſome mark in the diſk very near to its edge towards which I judged the line *on* ſhould be directed; then putting on the eye-piece which carries the micrometer I

took the diſtance according to this judgment as well as I could.

The third method I took was the following, which, indeed, I look upon as the beſt of all, and which I therefore moſt frequently put in practice. I took a view of ſome neighbouring ſhades of rocks or mountains, if there happened to be any near, and directed the meaſure of the micrometer by them, as they plainly pointed out the direction of the illuminating ray; or, which is the ſame thing, indicated the line perpendicular to a line joining the cuſps.

Mons Leucopetra was meaſured by this laſt method, which circumſtance I have mentioned in my obſervations of the 12th of March, where I ſaw the whole rock and its higheſt point, as well as the whole ſhade, and its laſt termination, upon very even ground, at the ſame time that I directed my micrometer in that line, to take the projection *on*, of the above mentioned mountain.

Sometimes I compared together a meaſure taken in the direction *on*, and one taken in the direction *nn*; but as moſt of my obſervations were made upon mountains not ſituated near the cuſps or limb of the Moon, I never found ſo much difference between theſe two meaſures, that it could have occaſioned any very material error, if I had intirely neglected it.

By the nature of the ellipſis it will appear, that, when we do not come too near the limb or cuſps of the Moon, a tangent drawn to a point in the curve of illumination will ſeldom make with the ſubtangent an angle that exceeds (or is ſo much as) 26°; and in all ſuch caſes the error that can ariſe from taking the line *rn* inſtead of *on* will be leſs than the tenth part of the whole meaſure: but, if the angle the tangent makes with the ſubtangent is only about 18°, the error will be leſs than a 20th part; and all the meaſures I have taken, I believe, will be found to be much within theſe laſt-mentioned limits. From this conſideration it will appear, that if I had not been aware of this circumſtance, my obſervations would ſtill be ſufficiently accurate to diſprove the uſually aſſigned great height of the lunar mountains; but as I took all the precaution the ſituation of each mountain would afford, by uſing any one of the above mentioned three methods, which ſuited beſt, I believe there can hardly be a poſſibility of any error that ſhould amount to a 40th part of the whole height of any mountain I have meaſured.

The figure ABCD (fig. 5.) contained by the diameter AB, the arch CD, and the two curve AD, BC, ſhews in what portion of the Moon's ſemi-diſk we may ſafely meaſure the line *rn*, inſtead of *on*, without being liable

to ſo great an error as one tenth part of the whole, and the figure A B *e d* contains that part wherein the meaſure *r n* being taken inſtead of *o n*, the error will be leſs than the 20th part of the whole meaſure. In a portion ſomething more confined the error will ſoon vaniſh, ſo that the difference may be ſafely neglected intirely. Thus in the ſpace A B *x y* the error cannot amount to a hundredth part. Theſe figures may be conſtructed by taking the ſeveral points D, *d*, *y*, and C, *c*, *x*, 26°, 18°, 8°, reſpectively, from the vertex, the curves A D, A *d*, A *y*, B C, B *c*, B *x*, being the loci of thoſe points of the tangents which touch the ſeveral ellipſes of illumination that may be contained in the ſemi-diſk of the Moon, when theſe tangents make thoſe ſeveral angles of 26°, 18°, 8°, with their ſubtangents.

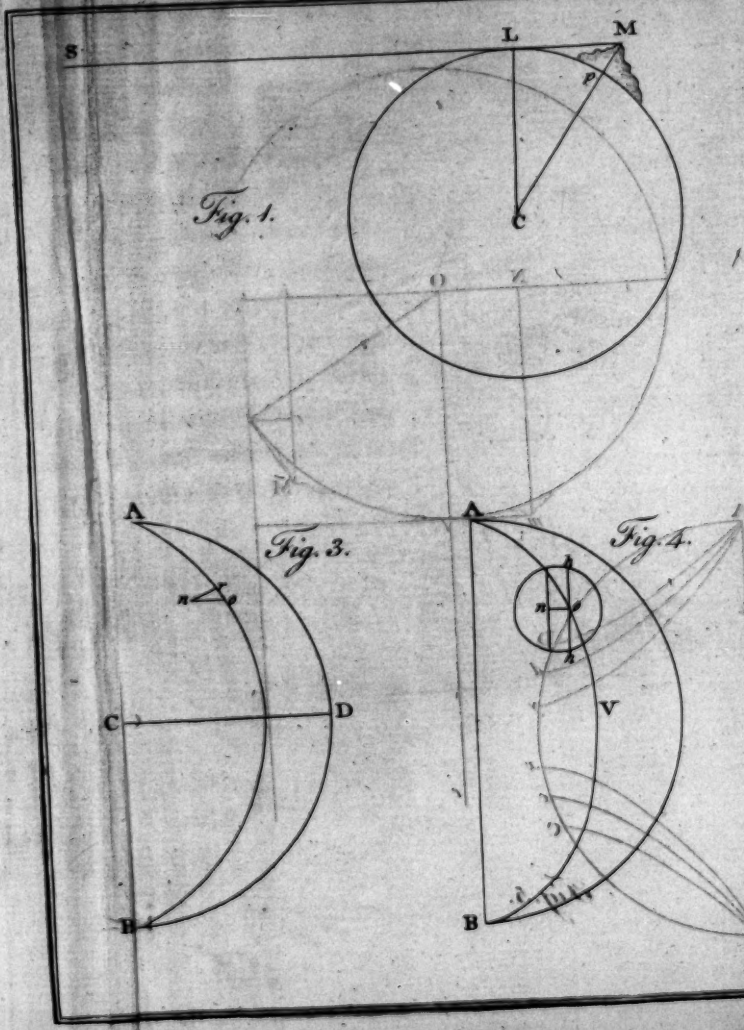
S
L
M
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Fig. 1.
O
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Fig. 3.
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Fig. 4.
n
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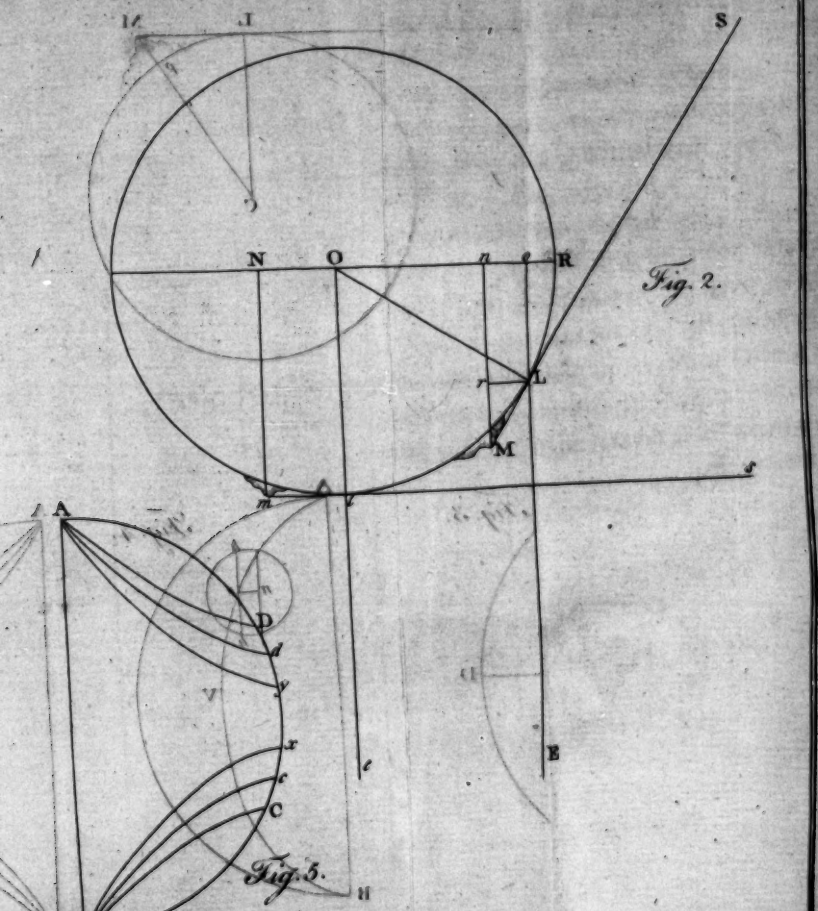

Fig. 2.

Fig. 3.

XXX. *Account of an extraordinary Pheaſant.* By *Mr.* John Hunter, *F. R. S.*

Read June 1, 1780.

EVERY deviation from that original form and ſtructure which gives the diſtinguiſhing character to the productions of nature may not improperly be called monſtrous. According to this acceptation of the term, the variety of monſters will be found to be infinite. As far as my knowledge has extended, there is not a ſingle ſpecies of animal, nay there is not a ſingle part of an animal body, which is not ſubject to extraordinary formation. Nor does this appear to be a matter of mere chance; for it may be obſerved, that each animal has a diſpoſition to deviate from nature in a manner peculiar to itſelf. It is likewiſe worthy of remark, that each ſpecies of animal is diſpoſed to have the ſame kinds of ſupernumerary parts, and nearly the ſame kinds of defects; but every part is not, perhaps, ſubject to a great variety of forms, each part of each ſpecies having its monſtrous

form, as it were, originally impreſſed on it by the hand of nature.

It is well known, that there are many orders of animals which have the two parts, deſigned for the purpoſe of generation, different in the ſame ſpecies, and which are thus divided into male and female: but this is not the only diſtinguiſhing mark in many genera of animals; in the greateſt part of animals, the male being diſtinguiſhed from the female by various marks.

A lion is different from a lioneſs; a cock from a hen, &c.; particularly the voice in many animals of the ſame genus is different: ſuch I ſhall call ſecondary properties, which take place only in parts that are neither eſſential to life nor generation, and which do not take place till towards the age of maturity. However, this diſtinction of ſexes, by any other marks than the difference of the parts of generation, is not obſervable in every order of animals; for inſtance, there is very little difference between a dog and a bitch, excluſive of the parts of generation.

In thoſe orders of animals, which are compoſed of diſtinct ſexes, we may obſerve, the genital organs not only ſubject to a mal-conformation, ſimilar to a mal-conformation in any other part of the animal; but we may likewiſe ſometimes obſerve an attempt to unite the

two

two parts in one animal body, producing an animal called an unnatural hermaphrodite[a].

It is my intention at preſent to extend my inquiry on this ſubject no farther than as to what relates to that reſemblance which one ſex bears to that of another in thoſe diſtinguiſhing properties which I term ſecondary.

The common claſs of unnatural hermaphrodites appear to be governed by certain laws, by which ſuch an extraordinary formation of parts is effected; for it is obſervable, that theſe deviations obtain through whole ſpecies of animals preciſely in the ſame manner. I have given an account of the Free-martin in a paper already preſented to this Society. This hermaphrodite exhibits a mixture of the two parts of generation in the ſame animal.

But we find, however, that there is often a change of the ſecondary properties of one ſex into another, the female now and then aſſuming the peculiarities of the male with reſpect to the ſcondary properties; and I may obſerve, that ſome claſſes are more ſubject to this than others; a ſingular example of which is to be the ſubject of the following paper.

Here I beg leave to premiſe, that in all animals of no diſtinct ſex, there is no alteration taking place in their

(*a*) Vide Phil. Tranſ. vol. LXIX. part I.

form

form when at the age of maturity, which I have obſerved not to be the caſe with moſt of thoſe animals which are of diſtinct ſexes, for in ſuch the ſecondary marks of diſtinction are exhibited at certain periods of life.

There is no ſex of any animal whatever that has any peculiarity in ſhape when born, or when young; but moſt of thoſe animals, which are of diſtinct ſexes, have peculiarities towards the age of maturity. The male at this time loſes that reſemblance which he before bore to the female in various ſecondary properties, excluſive of what relates to the organs of generation (*b*). That it is the male who at this time recedes from the female in this reſpect is evident. Every female, juſt at the age of maturity, is more like the young of the ſame ſpecies than the male is obſerved to be; and if the male is deprived of his teſtes when young, he retains more of the original form, and therefore is more ſimilar to the female.

From hence it might be ſuppoſed, that the female character contains more truly the ſpecific properties of the animal than the male; but the true character of every animal is that which is in both ſexes, *viz.* a natural hermaphrodite, or an animal of neither ſex, *viz.* a

(*b*) This is not common to all animals of diſtinct ſexes; for in the fiſh there is no great difference, nor in many inſects, nor in dogs, as has been already obſerved; it appears to be moſt ſo in birds.

caſtrated

caſtrated male or ſpayed female. Of the firſt we may inſtance the ſnail, which is of both ſexes, and has but one character, but that of the joint character of both ſexes.

But where the ſexes are ſeparate, and in ſpecies which have two characters, neither of them can be called the true one; the true diſtinguiſhing properties being thoſe peculiar to neither ſex, which are found in the caſtrated male, the ſpayed female, or the monſtrous hermaphrodite. That this is the diſtinct character of ſuch animals is evident, for the caſtrated male and the ſpayed female have but one ſet of properties between them; and when I treated of the Free-martin, which is a monſtrous hermaphrodite, I obſerved, that it was more like the ox than the cow or bull, ſo that the double ſex which contains the true character of every animal is imitated when made of no ſex by art, and by that means gives us the true properties of the ſpecies.

In the Free-martin the character ariſes from a mixture of ſexes; but in ſome animals, which have ſecondary principles peculiar to the two ſexes, we have a deviation from all thoſe general rules. We have in ſome a change of thoſe ſecondary characters, the perfect female with reſpect to the parts of generation, aſſuming more or leſs of the ſecondary character of the male.

This, however, does not appear to be a principle the action of which takes place at the first formation of the animal, so as to grow up with it, but appears to be one of those actions which take place, perhaps, at certain periods of life, similar to many common and natural phenomena; like to what is observed of the horns of the stag, which differ at different ages; or to the mane of the lion, which does not grow till after his fifth year, &c.

This change has been observed in some of the bird tribe, but principally in the common pheasant.

It is remarked by those who are conversant with this bird, when wild, that there appears every now and then a hen pheasant with the feathers of the cock; and all that they have decided on this subject is, that this animal does not breed, and that its spurs do not grow. Some years ago one of these was sent to Dr. HUNTER, who gave me leave to examine it. I found, upon examination, that it had all the parts of the female peculiar to that bird. This specimen is still preserved in Dr. HUNTER's Museum.

Dr. PITCAIRN, having lately received a pheasant of this kind from Sir THOMAS HARRIS, exhibited it as a curiosity to Mr. BANKS and Dr. SOLANDER. I happened to be then present, and was desired to examine the bird. The following is the result of my examination.

I found

I found the parts of generation to be truly female: they were as perfect as in any hen pheaſant that is not in the leaſt prepared for laying eggs. There were both the ovaria and the ovi-duct.

As theſe obſervations have hitherto been principally made upon birds that are wild, little more can be known of them; but from what happened to a hen pheaſant belonging to a lady well known to the Preſident, it ſhould ſeem probable, that this character originates from a change at a late period of the animal's life, and does not originally grow up with it. This lady for ſome time had bred pheaſants, and had paid particular attention to them. One of her hens, after having produced ſeveral broods, moulted, and the ſucceeding feathers were thoſe of a cock. This animal was never afterwarde impregnated. Hence it is moſt probable, that all thoſe hen pheaſants which are found wild, and have the feathers of the cock, were formerly perfect hens, but that they are now changed by age, and perhaps by certain conſtitutional circumſtances.

This change of feather in hen pheaſants, although perhaps more common in them than in any other bird, yet is not abſolutely peculiar to them; for we have a well atteſted inſtance of the ſame phenomenon in a pea-

hen, nearly under the ſame circumſtances as have been above deſcribed.

Lady TYNTE had a favourite pyed pea-hen, which had produced chickens eight ſeveral times; having moulted when ſhe was about eleven years old, ſhe aſtoniſhed the lady and her family by the feathers peculiar to the other ſex, and appearing like a pyed peacock. In this proceſs the tail, which was ſimilar to that of a cock, firſt appeared after moulting. In the following year, ſhe moulted again, and produced the ſame feathers. In the third year ſhe did the ſame: at the ſame time ſhe had ſpurs ſimilar to thoſe of a cock. She died in the following winter during the hard froſt, namely, in the winter $177\frac{3}{4}$. She never bred after this change in her plumage. This bird is now preſerved in the Muſeum of Sir ASHTON LEVER [*b*].

From what has been related of theſe two birds, may it not reaſonably be inferred, that it ſeems probable, that all thoſe wild pheaſants of the female ſex, which are

(*b*). It might be ſuppoſed, that this bird was really a cock which had been changed for a hen; but the following facts put this matter beyond a doubt. Firſt, there was no other pyed pea-fowl in the country. Secondly, the hen had knobs on her toes, which were the ſame after her change. Thirdly, ſhe was as ſmall after the change as before, therefore too ſmall for a cock. Fourthly, ſhe was a favourite bird, and was generally fed by the lady, and uſed to come for her meat, which ſhe ſtill continued to do after the change in the feathers.

found

found with the feathers of the cock, had changed the nature of their feathers, particularly at a certain age?

If this idea be just, it shews, that there is a disposition in the female to come nearer and nearer to the male, at least in the secondary properties; or it may rather be said, that the female is later in producing this change than the male is; for it has already been observed, that both sexes when young differ not from each other in these respects, but that the male appears to be the one that by degrees separates itself from the female in its secondary properties.

XXXI. *A Letter to* Joſeph Banks, *Eſq. Preſident of the* Royal Society, *&c. from* Daniel-Peter Layard, *M. D. Fellow of the Royal Societies of* London, Antiquaries, *and* Gottingen, *&c. relative to the Diſtemper among the horned Cattle.*

Read June 15, 1780.

SIR,

YOUR noble predeceſſor, the late Earl of MACCLESFIELD, thought that every information relative to the aſcertaining the nature of the diſtemper among the horned cattle, and to confirm the utility of inoculation, was worthy attention, and the notice of the Royal Society.

His lordſhip was pleaſed, in 1757, to lay before the Society my letter, which is inſerted in vol. L. of the Philoſophical Tranſactions; and now, SIR, to a worthy and reſpectable ſucceſſor, whoſe life, health, and fortune, have been moſt generouſly adventured for the promoting natural knowledge, I addreſs this, to impart the reſult of a long and ſtrict inquiry.

In conſequence of the eſſay which I publiſhed in 1756, I was called upon in 1769, by government, to aſſiſt with my advice towards the ſtopping the progreſs of the contagious diſtemper among the cattle, which had broken out in Hampſhire: and by mere accident I diſcovered how the infection was brought from Holland to London, and was conveyed into that county.

Speedily and effectually to extirpate the calamity, no aſſiſtance was permitted to viſit the infected villages, leſt the farmers ſhould be induced to prolong the illneſs, by attempting to cure their cattle; but poſitive orders were iſſued that all the cattle ſhould be killed and buried properly, by which vigorous and ſalutary directions the diſtemper ceaſed intirely in a ſhort time.

The ſame acts of parliament and orders of council, to kill the cattle and bury them deep, ſucceeded alſo ſoon after in North Britain; and to the former acts and orders iſſued in his late Majeſty King GEORGE the ſecond's reign, theſe alterations were made: to order that the infected cattle ſhould be killed, without effuſion of blood, by ſtrangling; the hides to be neither cut or ſlaſhed; but the carcaſſes buried whole; and that all the fodder, litter, excrement, &c. ſhould be buried, inſtead of being burned.

Since

Since that time the contagious diſtemper has been brought twice into Eſſex, and once into Suffolk, from Holland, and as often ſtopped by the ſame means.

His Majeſty having moſt graciouſly been pleaſed in April 1770 to appoint me to hold a foreign correſpondence, the orders and regulations which had happily ſucceeded in Great Britain were communicated to the Dutch, the Flemiſh, and the French, and copies of all papers delivered to Baron NOLCKEN, the Swediſh miniſter. In Flanders, and Picardy in France, the ſyſtem of killing was adopted, and ſucceeded. Afterwards in 1774, when the ſame contagion was carried into the South of France from Holland through Bourdeaux, many attempts having failed to effect a cure, the devaſtation was at laſt ſtopped by no other means than by killing the cattle, as in Great Britain. And here I beg leave to obſerve, that Monſ. VICQ D'AZYR, in his *Expoſé des Moyens Curatifs et Préſervatifs contre les Maladies peſtilentielles, des Betes à Corne*, publiſhed by authority at Paris in 1776, ſays, p. 577, "That the ſalutary effects "of the precautions taken in the Auſtrian Low Coun- "tries had excited the attention of the Engliſh, who by the "ſame means got rid of the ſame calamity. They have "exactly and ſcrupulouſly tranſlated and put into exe- "cution the edicts iſſued from the Juntos of Ghent and

"Bruſſels, and their undertaking has been crowned with "the moſt complete ſucceſs." Monſ. VICQ. D'AZYR was miſinformed; for, on the contrary, the late Mr. Conſul IRVINE tranſmitted the acts of parliament, the orders of council, and my papers, containing every neceſſary information, to a member of the Junto of Ghent, whence they were ſent to the government at Bruſſels; and it was a long while before the Juntos could be prevailed upon to adopt the ſyſtem of killing, as they called it. It originated in England in 1747; and it is certain, that the Court of Vienna knew fully the obligations which the Auſtrian Netherlands had to the Britiſh government, whoſe orders and regulations had been implicitly followed, and which Monſ. VICQ. D'AZYR, ſays, p. 585, "He had modified and adapted to the rules of French "government."

In Flanders the infection was alſo prevented from ſpreading a ſecond time by the ſame method of proceeding; but unfortunately in Holland the cattle continue to be expoſed to the ſame diſeaſe. The half-yearly returns which have been regularly ſent me contain melancholy accounts of the ſevere loſs of cattle; ſometimes the whole have periſhed; at other times two-thirds have died; and generally above half fell when the ſickneſs was leſs violent. In a country where the illneſs is become

general, and conſtantly raging more or leſs, where the ſyſtem of killing the cattle cannot now be thought of, and where inoculation has met with ſo many opponents of all ranks, there can be no other hope of getting rid of the calamity than by admitting into the United Provinces no other cattle than ſuch which are found, or recovered from the infection [a].

I ſhall not trouble you, SIR, with the returns from Holland, or the tables of inoculation in Denmark, which would too much increaſe the length of this letter, but only mention, that in Denmark, where the contagious diſtemper is become naturalized and general, the Daniſh government have not only wiſely adopted the orders and regulations iſſued in Great Britain, but have with unwearied application purſued the practice of inoculation. Count BERNSDORFF and Dr. STRUENSEE had all the neceſſary inſtructions, books, and papers, delivered to them by me, when the King of Denmark was in England; and I am aſſured by DANIEL DELAVAL, Eſq. lately his Majeſty's envoy-extraordinary at that Court, that inoculation is approved, recommended, and by authority eſtabliſhed. Even in the firſt three years that inoculation was practiſed, of near three hundred head of cattle which were inoculated in a Daniſh iſland, not a ſixth

(a) By the laſt half-yearly return from Holland, the number of infected cattle was ſo ſmall, that it was hoped no further return to the States would be neceſſary.

part

part were loſt, notwithſtanding the many diſadvantages which unavoidably occurred.

Profeſſor CAMPER had before attempted to introduce inoculation in Holland; but the learned Profeſſor's abilities, diligence, and perſeverance, were ſo much counteracted by the obſtinacy and interruption of the peaſants, the badneſs of the ſituation, and inclemency of the weather, that out of 112 only 41 recovered; and yet that number is full ſufficient to prove his opinion of the diſeaſe, and of the uſe of inoculation.

Application was made, in 1770, to the Lord Preſident of the Council by a famous inoculator, for leave to take matter from the infected beaſts in Hampſhire, and to inoculate the cattle in the Southern and Weſtern counties of England: on a repreſentation to his lordſhip, that by ſuch an operation the contagion would not only be introduced in thoſe counties where it had not yet appeared, but alſo might ſpread the ſickneſs, ſo as to become general all over the kingdom as before, a poſitive and ſtrict injunction was given to drop the intention; eſpecially as by killing the cattle there was no doubt of extirpating the contagion out of Hampſhire. The inoculator therefore made no attempt.

According to the ſeveral prejudices of different countries, various opinions have ariſen of the nature of this

ſickneſs. Such as are averſe to inoculation have obſtinately refuſed to acknowledge it was ſimilar to the ſmall-pox in the human body, and have very idly aſſerted, that the only intention of declaring this contagion to be a ſort of ſmall-pox was purpoſely, and with no other view than to promote inoculation for the ſmall-pox. Others have as poſitively declared it to be a peſtilential putrid fever, owing to a corrupted atmoſphere, and ariſing from infected paſtures; but unfortunately for the ſupporters of this opinion, while the contagious diſtemper raged with the utmoſt violence on the coaſts of Friſeland, North and South Holland, Zealand, and Flanders, there was not the leaſt appearance of it on the Engliſh Coaſt from the North Foreland to the Humber, although the coaſt and climate are the ſame.

I ſhall not dwell on Mr. TURBERVILLE NEEDHAM's eloquent diſcourſe read at Bruſſels, ſince he muſt have been convinced, when he came to England in 1776, that the illneſs was of another ſort than he imagined; for ſuch a proof of the inefficacy of ſalt recommended by him as an antiſeptic in this diſeaſe has been given as is poſitive and deciſive; namely, that in Scania, a province in Sweden, where it is cuſtomary to place a large piece of rock ſalt, called *ſalt-ſtein*, for the cattle houſed in winter to lick, that they may be urged to drink, all the cattle in

that

that province were seized with the contagious distemper, and not one out-lived it. Monſ. PAULET, in his *Recherches ſur les Maladies Epizootiques*, vol. II. p. 25, 26. Paris 1776, has ſufficiently exploded Mr. NEEDHAM's opinion.

M. BERGIUS had inſiſted, that the contagion was not of the exanthematous ſort, and therefore inoculation muſt be of no uſe; but this opinion was alſo fully refuted by the late Profeſſor ERXLEBEN of Göttingen, in his learned oration on the 20 of October, 1770.

From every information, domeſtic or foreign, and comparing the ſeveral opinions, experience and obſervation plainly and completely determine the diſpute. The diſeaſe among the horned cattle, ſo fatal in many countries, is not endemial or natural to Europe, although it is become ſo in Denmark, from ſpreading all over the Daniſh dominions, and its long continuance in that kingdom. It is an eruptive fever of the variolous kind (*b*), and notwithſtanding the exanthemata, or puſtules, may have been frequently overlooked, yet none ever recovered without more or leſs eruption or critical abſceſſes; but theſe differ from the peſtilential ſort; no otherways ſimi-

(*b*) In a letter from Monſ. VICQ D'AZYR to Dr. LAYARD, dated Paris, Auguſt 28, 1780, is the following declaration: "Il me paroit comme à vous "que c'eſt toujours la meme maladie qui a regné depuis 1711, et qu'elle a "de grands rapports avec l'eruption varioleuſe."

lar

lar to the plague, but, like unto the ſmall pox, it is communicated by contact, by the air conveying the effluvia, which alſo lodge in many ſubſtances, and are thereby carried to very diſtant places. Unlike other peſtilential, putrid, or malignant fevers, it bears all the characteriſtic ſymptoms, progreſs, criſis, and event of the ſmall-pox; and, whether received by contagion or inoculation, has the ſame appearances, ſtages, and determination, except more favourably by inoculation, and with this diſtinctive and deciſive property, that a beaſt having had the ſickneſs, naturally or artificially, never has it a ſecond time.

Thus, Sir, I have endeavoured to lay before you and the Royal Society, the reſult of my inquiries, experiments, obſervations, and correſpondence, concerning this calamitous ſickneſs, which, from my ſituation in Huntingdonſhire in 1756, it fell to my lot to inveſtigate.

His Majeſty's paternal care recommended from the Throne in 1770, and ordered every attention to be given to free his ſubjects from the impending calamity, which had been felt ſo ſeverely in former years. The great Council of the Nation gave the ſtrongeſt aſſurances of their readineſs to ſupport and aſſiſt the Royal commands; and the moſt ſalutary and effectual directions and orders, which originated in Great Britain were humanely and generouſly tranſmitted, by a conſtant correſpondence in

the courſe of above ten years, to every diſtreſſed ſtate abroad, who alſo have enjoyed the ſame ſucceſs.

You will allow me, SIR, I hope, to expreſs the peculiar ſatisfaction I feel as an Engliſhman, that my endeavours in this public ſervice are honoured with the approbation of our moſt gracious Sovereign, and the goodwill of my native country.

And happy in this opportunity I requeſt you, SIR, to accept of my warmeſt wiſhes that you may long fill the high office of Preſident of the Royal Society, with health to purſue your philoſophical reſearches; and may enjoy from the moſt reſpectable and liberal Society in the world, the juſt and due tribute of their hearty acknowledgements for having their inſtitution, honour, and intereſt, ſo much at heart.

I have the honour, &c.

Lower Brook Street,
April 8, 1780.

XXXII. *An Inveſtigation of the Principles of progreſſive and rotatory Motion.* *By the Rev.* S. Vince, *A. M. of* Sidney College, Cambridge. *Communicated by* George Atwood, *A. M. F. R. S.*

Read June 15, 1780.

THE communication of motion from impact is well known to conſtitute a conſiderable part of that branch of natural philoſophy called mechanics; and as all our enquiries therein are directed, either to aſſiſt us in thoſe operations which add to the conveniences of life, or to explain, for the ſatisfaction of the mind, thoſe changes which we daily ſee ariſe from the effects of bodies on each other, it might naturally have been expected that the attention of philoſophers would have been engaged, firſt in the inveſtigation of ſuch caſes as moſt frequently occur from the accidental action of one body on another, before they had proceeded to others leſs obvious. A little conſideration will convince any one how ſeldom it happens, in the colliſion of two bodies, that their centers of

of gravity and point of contact lie in the line of direction of the ſtriking body, yet few writers on mechanics have extended their enquiries any further than this ſimple caſe. It muſt however be acknowledged, that the action of bodies on each other, in directions *not* paſſing through their center of gravity, affords a ſubject at leaſt curious in ſpeculation; for my own part, I have little doubt but that it might be rendered extremely uſeful to the practical mechanic. I. BERNOULLI was the firſt who publiſhed any thing on this ſubject. He found the point about which a body at reſt would begin to revolve when ſtruck by another body, obſerving however that D. BERNOULLI had alſo diſcovered the ſame: he has alſo mentioned the curve deſcribed by that point in the progreſſive motion of the body, and has directed a method of enquiry by which the velocities of the bodies may be found after the ſtroke, which comprehends all he has done on the ſubject. Two years afterwards D. BERNOULLI publiſhed a paper on progreſſive and rotatory motion, containing nothing more than what I. BERNOULLI had before given us, and, what is a little extraordinary, ſays in the introduction, *de tali quidem percuſſione nihil adhuc, quantum ſcio, publici juris factum fuit ab iis, qui de motu corporum a percuſſione egerunt.* EULER has alſo inveſtigated the velocities of the

bodies after impact in a manner ſomewhat different, but has rendered it much more intricate by a fluxional calculus. To any one, however, who attentively conſiders the ſubject, the theory muſt ſtill appear to be extremely imperfect, as, independent of principles not more ſelf-evident than the propoſitions they are intended to demonſtrate, which both I. and D. BERNOULLI have aſſumed in their inveſtigations, a great variety of other circumſtances equally intereſting naturally ariſe in an enquiry into this matter, circumſtances abſolutely neceſſary towards underſtanding the principles of the motion of the bodies after impact. This induced me to conſider the ſubject with ſome attention, and preſuming that I have not been altogether unſucceſsful in my endeavours to render the theory more perfect, I determined to lay the reſult of my enquiries before the Royal Society. I thought it expedient, for the ſake of perſpicuity, to divide the whole into diſtinct Propoſitions; and as the moſt ſimple caſes are beſt underſtood, I have firſt conſidered the caſe of the action of a body on a lever having a corpuſcle at each end: and I was the more induced to treat the ſubject in this manner, as moſt of the principles can be immediately applied to any number of corpuſcles, in conſequence of which the general inveſtigations are rendered more eaſy and ſatisfactory.

PROP

PROP. I.

Let A *and* B *be two indefinitely ſmall bodies connected by a lever void of gravity, and ſuppoſe a force to act at any point* D *perpendicularly to the lever, to find the point about which the bodies will begin to revolve.*

From the property of the lever, the effect of the force acting at D (fig. 1.) on the body A is to the effect on B as BD : AD; hence the ratio of the ſpaces A*m*, B*n*, deſcribed by the bodies A and B in the firſt inſtant of their motion, will be as $\frac{BD}{A} : \frac{AD}{B}$; join *mn*, and if neceſſary produce that line and AB to meet in C, which will manifeſtly be the point about which the bodies begin to revolve. Hence from ſimilar figures BC : AC :: $\frac{AD}{B}$(∝B*n*) : $\frac{BD}{A}$ (∝A*m*) :: A×AD : B×BD, or DC−DB : AD + DC :: A×AD : B×BD, and conſequently $DC = \frac{A \times AD^2 + B \times BD^2}{B \times BD - A \times AD}$, and therefore D is the center of percuſſion or oſcillation to the point of ſuſpenſion C.

Cor. 1. Hence, whatever be the magnitude of the ſtroke at D, the point C will remain the ſame.

Cor. 2. If the force acts at the center of gravity G, the bodies will have no circular motion, for in this caſe B×BD−A×AD=0, and therefore DC becomes infinite.

Cor. 3. If the force acts at one of the bodies, the center of rotation C will coincide with the other body.

Cor. 4. If the lever had been in motion before the ſtroke, the point C, at the inſtant of the ſtroke, would not have been diſturbed.

PROP. II.

Let a given quantity of motion be communicated to the lever at D, *to determine the velocity of the center of gravity* G.

The ſpace A*m*, deſcribed by the body A in the firſt inſtant of motion, is as $\frac{DB}{A}$: now $CG = CD - DG = CD - AG + AD = \frac{A \times AD^2 + B \times BD^2}{B \times BD - A \times AD} - AG + AD = \frac{B \times BD \times BG + A \times AD \times AG}{B \times BD - A \times AD}$; alſo $CA = CD + DA = \frac{A \times AD^2 + B \times BD^2}{B \times BD - A \times AD} + DA = \frac{B \times BD \times AB}{B \times BD - A \times AD}$; hence we have $\frac{B \times BD \times AB}{B \times BD - A \times AD}(AC) : \frac{BD}{A}(\propto mA) :: \frac{B \times BD \times GB + A \times AD \times AG}{B \times BD - A \times AD}(CG) : \frac{B \times BD \times GB + A \times AD \times AG}{A \times B \times AB} \propto G\omega$ the velocity of the center of gravity: hence if the motion be communicated at G, the velocity becomes as $\frac{B \times GB^2 + A \times AG^2}{A \times B \times AB}$.

Let now the motion, which is ſuppoſed to be actually communicated to the rod at D, be equivalent to the motion of a body whoſe magnitude is G, and moving with a velocity v; then if that motion be communicated at G, the velocity of the center of gravity is well known to be

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$= \frac{G \times v}{A+B}$; hence $\frac{B \times BG^2 + A \times AG^2}{A \times B \times AB} : \frac{B \times BD \times BG + A \times AD \times AG}{A \times B \times AB} :: \frac{G \times v}{A+B} : \frac{G \times v}{A+B} \times \frac{B \times BG \times BD + A \times AD \times AG}{B \times BG^2 + A \times AG^2}$ = the velocity of the center of gravity, when the ſame motion is actually communicated to any point D. Now BD = BG + GD, and AD = AG − GD; hence $B \times BG \times BD + A \times AD \times AG = B \times BG^2 + A \times AG^2 + GD \times \overline{B \times BG - A \times AG}$ = (becauſe $B \times BG - A \times AG = 0$) $B \times BG^2 + A \times AG^2$; conſequently the velocity becomes $\frac{G \times v}{A+B}$; and hence the center of gravity moves with the ſame velocity, wherever the motion is communicated.

PROP. III.

Let a given elaſtic body P, *moving with a given velocity, be ſuppoſed to ſtrike the lever at the point* D *in a direction perpendicular to it; to determine the velocity of the center of gravity* G *after the ſtroke.*

Suppoſe firſt the body to be non-elaſtic, and let v be the velocity of the center of gravity after the ſtroke upon that ſuppoſition, and V the velocity of the ſtriking body: then CG : CD :: v : $\frac{v \times CD}{CG}$ = the velocity of the point D after the ſtroke, or of the body P; for the ſame reaſon $\frac{v \times CA}{CG}$ and $\frac{v \times CB}{CG}$ equal the velocities of A and B reſpectively. Now, becauſe in revolving bodies, the

the momenta, ariſing from the magnitude of the bodies, their diſtance from the center of rotation and velocity conjointly, remain the ſame after the ſtroke as before, we ſhall have $P \times V \times DC = \frac{v \times CD^2 \times P}{CG} + \frac{v \times CA^2 \times A}{CG} + \frac{v \times CB^2 \times B}{CG}$, and therefore $v = \frac{P \times V \times DC \times CG}{P \times DC^2 + A \times AC^2 + B \times BC^2} = \frac{P \times V \times CG}{A+B \times CG + P \times DC}$; hence if P be ſuppoſed an elaſtic body, we ſhall have $\frac{2 \times P \times V \times CG}{A+B \times CG + P \times DC}$ for the velocity of the center of gravity after the ſtroke, *in ipſo motûs initio*.

PROP. IV.

To determine the motion of the bodies after the firſt inſtant, or when they are left to move freely by themſelves.

The writers on mechanics, from conſidering the equality of motion on each ſide the center of gravity, when a body revolves about that point, have inferred, that if a body had a projectile as well as a circular motion communicated to it, the center of gravity would continue to move in a right line, as that point would not be diſturbed by the rotatory motion: yet, as, in the caſe we are now conſidering, the bodies begin to revolve about a different center, it may be proper to examine more accurately into this matter, and to ſhew from what

what principle it is that the motion of the center of gravity is preſerved in a right line.

Let a motion perpendicular to the rod be communicated to A (fig. 2.) and then by Cor. 3. Prop. I. B will not be diſturbed by ſuch an action; and A will in the firſt inſtant have a tendency to revolve about B as a center, and would actually deſcribe the arc AH, if the body B were fixed: let the angle ABH be ſuppoſed infinitely ſmall, and let GK be the arc, the center of gravity would have deſcribed, and draw the tangents AF, G*g* to the arcs AH, G*g* reſpectively. Now, if A could have moved freely, it would (becauſe AF = AH) have deſcribed AF in the ſame time the arc AH was deſcribed, upon ſuppoſition that B was fixed; for the radius BA being perpendicular to the circular arc AH, the force of the lever could have no efficacy to accelerate or retard the motion of A in the arc AH, and therefore the velocity in that arc is the ſame as it would have been if it had moved freely in the tangent: hence HF is that ſpace through which the centrifugal force of A would have carried that body, could it have moved freely; but as A is connected to B by means of the lever, it is manifeſt that the ſame force which would have carried A from H to F in the direction of the lever, will, when it has both bodies to move, carry it over a ſpace which is to FH as A : A+B,

or

or as Bg : BH, or as gK : FH; hence that ſpace, or the ſpace through which the centrifugal force of A will draw the lever in the direction BH, is equal to K*g*; that is, the point K, which is the center of gravity of A and B, will be found at *g*, and conſequently the center of gravity has preſerved its motion uniform in the right line G*g*, inaſmuch as the centrifugal force, acting perpendicularly to the direction of the center of gravity, can neither accelerate or retard its motion. In the ſame manner it may be proved, that the motion of the center of gravity is continued uniform in the ſame right line, whatever be the poſition of the lever. Moreover, as the centrifugal force acts in the direction of the lever, it cannot alter its angular velocity, which will therefore remain as *in ipſo motûs initio*. If now we ſuppoſe that to the force impreſſed upon A, two other equal accelerative forces be communicated to A and B at the ſame time, it is evident that no alteration can ariſe from the actions of the bodies on each other; and the caſe will then be ſimilar to the motion of the bodies, ſuppoſing a ſingle force had been impreſſed at any point D. The like method of reaſoning may be extended to any number of bodies.

The ſame thing may alſo eaſily be demonſtrated in the following manner. The centrifugal forces of A

and B (fig. 1.) are reſpectively $A \times AC$ and $B \times BC$; alſo the centrifugal force of the point G, conſidering it as having both bodies to move in the direction of the rod, is $\overline{A+B} \times GC$, but from mechanics $A \times AC + B \times BC = \overline{A+B} \times GC$: hence the centrifugal forces of the bodies A and B give the center of gravity a centrifugal force equivalent to its own centrifugal force, which, as the latter would cauſe that center to move in the tangent Gg, the lever not being fixed at C, it is manifeſt that the former will cauſe the center of gravity to continue its motion in the ſame direction.

That this motion of the lever, in a direction from the center C, is the only motion which is communicated to it from the effect of the bodies A and B is manifeſt from hence. The bodies begin to revolve freely about the point C, and conſequently if the point C had been fixed, the bodies would have moved on with a uniform angular velocity about C; if therefore we ſuppoſe the lever not to be fixed at C, as the efficacy of the centrifugal force which acts in the direction of the lever is now ſuffered to take place, and no new external force is impreſſed on either of the bodies, it is manifeſt, that if in the former caſe the bodies had no efficacy to diſturb the angular velocity of the lever, they cannot have any in the latter, conſequently the angular velocity, and from what

has been before proved, the uniform motion of the center of gravity in a right line, remain unaltered, after the commencement of the motion.

PROP. V.

In the time the bodies make one revolution, the center of gravity will move over a ſpace equal to the circumference of a circle whoſe radius is CG (*fig.* 1.)

From the laſt Propoſition, the angular velocity of the lever is continued uniform; hence the time of a revolution is juſt the ſame as if the point C were fixed, and the bodies were to continue to revolve about that point as a center, in which caſe the center of gravity G, in the time of a revolution, would evidently deſcribe the circumference of a circle whoſe radius is GC. This therefore is the ſpace the center of gravity deſcribes in a right line when the bodies move freely, for from the laſt Propoſition that center is carried uniformly forward with the ſame velocity.

Cor. 1. Hence if the magnitude of the force acting at D vary, the velocity of the center of gravity will vary in the ſame ratio as the angular velocity.

Cor. 2. Hence the point D may be found, where a force being applied, the bodies ſhall make one revolution,

tion, whilſt the center of gravity moves over any given ſpace (s): for let p = the periphery of a circle whoſe radius is unity, then $p : 1 :: s : \frac{s}{p}$ = the radius of a circle whoſe circumference is the ſpace to be paſſed over in the time of a revolution, and which muſt therefore, by the Propoſition, be equal to GC; the point C therefore being determined, D may be eaſily found, for from mechanics $CG \times DG$ is given; and from Cor. 3. Prop. I. when D comes to A, C will coincide with B, $\therefore CG \times GD = AG \times GB$, and conſequently $DG = \frac{AG \times GB}{CG}$.

PROP. VI.

To determine the time of one revolution, ſuppoſing every thing given as in Prop. III.

The point D being given, we have from Cor. 2. to the laſt Propoſition, $CG = \frac{AG \times GB}{DG}$; put w equal the circumference of a circle whoſe radius is CG, and it appears from the laſt Propoſition, that w is the ſpace the center of gravity paſſes over in the time of one revolution; hence, becauſe from Prop. IV. the center of gravity moves uniformly, we have by Prop. III. $\frac{s \times V \times P \times CG}{A+B \times CG+P \times DC}$

$: 1'' :: W : W \times \frac{2 \times V \times P \times CG}{A+B \times CG + P \times DC}$ = the time of one revolution.

Cor. Hence the angular velocity being inverſely as the time of a revolution, will vary as $\frac{A+B \times CG + P \times DC}{V \times P \times CG \times W}$.

PROP. VII.

The point C, *as the center of gravity moves forward, will deſcribe the common cycloid.*

From the deſcription of the common cycloid it appears, that the center of the generating circle paſſes over a ſpace equal to the circumference of that circle whilſt it makes one revolution. With the center G (fig. 3.) and radius GC, deſcribe the circle C*xy*, and draw CR, GW perpendicular to ABC, and let the circle C*xy* be ſuppoſed to revolve on the line CR; then will the center G move over a ſpace equal to the circumference of the circle C*xy* whilſt it makes one revolution, and the point C will deſcribe the common cycloid: but from Prop. V. the point G *will* move over a ſpace equal to the circumference of a circle whoſe radius is GC, whilſt the bodies, and conſequently GC, make one revolution; and hence the point C will deſcribe the ſame curve as before, that is, the common cycloid.

PROP.

PROP. VIII.

Let a motion be communicated to the lever obliquely, to determine the point about which the bodies begin to revolve.

Let FD (fig. 4.) repreſent the force communicating the motion at the point D, which reſolve into two others FH, HD, the former FH parallel to the lever, and the latter HD perpendicular to it. Let C be the point about which the bodies would have begun to revolve, had the force HD only acted, and which may be found by Prop. I: and ſuppoſe in this caſe *mgn* to have been the next poſition of the lever after the commencement of the motion, or that the bodies A, B, and center of gravity G, had been carried to *m*, *g* and *n* reſpectively. But as the force FH acts at the point D at the ſame time in the direction of the rod, if we take G*q* : G*g* as FH : HD, then whilſt the center of gravity would have moved from G to *g* in conſequence of the force HD, it will by means of the force FH be carried in the direction of the lever from G to *q*, and alſo every other point of the lever will be carried in the ſame direction with the ſame velocity; take therefore A*p* and B*r* each equal to G*q*, and complete the parallelograms A*a*, G*v* and B*b*, and the bodies A, B, and center of gravity G will, at the end of that time, be found

found at *a*, *b* and *w* reſpectively, and *awb* will be the poſition of the lever. Now it is evident, that C is not the point about which the bodies begin to revolve, for (conſidering the lever to be produced to C) that point muſt have moved over a ſpace C*c* equal to G*q*, when the lever is come into the poſition *awb*: draw CO perpendicular to CB, and GO perpendicular to G*w*, and O will be the center of rotation at the commencement of the motion. For conceive CO to be a lever, then the lever ABC has a circular motion about C, whilſt that point is moving from C to *c*, and conſequently the point O is carried forward in a direction parallel to C*c* by this motion; but as the lever CO is carried by a circular motion about C in a contrary direction, it is evident that that point of the lever CO muſt be at reſt where theſe two motions are equal, as they are in contrary directions. Now the velocity of C in the direction C*c* : velocity of G about C : : G*q* : G*g* : : (by ſim. triang.) CO : CG, and the velocity of the point G about C : velocity of the point O about C : : CG : CO; hence *ex æquo* the velocity of C in the direction of C*c*, or of O in the direction or parallel to C*c*, is equal to the velocity of the ſame point O in a contrary direction ariſing from its rotation about C, and conſequently O being a point at reſt, muſt be the center of rotation *in ipſo motûs initio*. Alſo, becauſe *ma* is equal

and

and parallel to *nb*, *ab* muſt be equal and parallel to *mn*, therefore the angular velocity is juſt the ſame as if the force FH had not acted. The center O of rotation at the beginning of the motion being thus determined, every thing relative to the motion of the bodies, after they are at liberty to move freely, may be determined as in the preceding Propoſitions.

Cor. 1. Hence it appears, that whatever be the magnitude or direction of the force communicating the motion, or the point at which it acts, the center of gravity will move in a line parallel to the direction of the force, for the triangles FHD, G*qw* being ſimilar, G*w* muſt be parallel to FD.

Cor. 2. The ſame is manifeſtly true for any number of bodies; for let (fig. 5.) E be a third body, and conceive it to be connected with the other two bodies A and B in their center of gravity G; then if FD repreſents the force acting at the point D, it is evident from the laſt Corol. and the ſecond Prop. that the center of gravity moves with the ſame velocity and in the ſame direction, as if the ſame motion had been communicated at G in a line RG parallel to FD, and that the center of gravity has the ſame velocity communicated to it, as if the two bodies had been placed at G; conceive therefore the bodies A and B to be placed at G, and let the force act at D, and

then from the laſt Corol. the center of gravity g, of the three bodies, will move in a line parallel to the direction of the force communicated. In the ſame manner it may be proved for any number of bodies.

SCHOLIUM.

The method here made uſe of to determine the point of rotation *in ipſo motûs initio*, when a ſingle force acts at any point D, may be applied, when any number of forces act at different points at the ſame time. For let (fig. 1.) α, β, γ, &c. repreſent the forces acting on the lever at the points D, E, F, &c. reſpectively; then from the ſame principles the effect of all the forces on A : the effect on B : : $\frac{\alpha}{AD} + \frac{\beta}{AE} + \frac{\gamma}{AF}$ + &c. : $\frac{\alpha}{BD} + \frac{\beta}{BE} + \frac{\gamma}{BF}$ + &c. which quantities put equal to P and Q reſpectively, and then $\frac{P}{A} : \frac{Q}{B}$: : Am : Bn : : AC : BC, from whence it appears, that (putting GC + GA = AC and GC − GB = BC) the diſtance GC = $\frac{A \times Q \times AG + B \times P \times BG}{B \times P - A \times Q}$. The ſame concluſion might have been deduced from this conſideration; that if any number of forces act on a lever, the effect on any point of that lever is juſt the ſame as if a force, equivalent to the ſum of theſe forces, had

had acted at their common center of gravity, find therefore their common center of gravity, and conceive a force equivalent to them all to be communicated to that point, and the Problem is reduced to the caſe of the firſt Propoſition. If any of the forces had acted on the oppoſite ſide of the lever, ſuch forces muſt have been conſidered as negative.

If there be any number of bodies placed on the lever, and a ſingle force acts at D, it will appear from the ſame principles that the point C, about which they begin to revolve, will be the point of ſuſpenſion to the center of percuſſion D; and the ſame concluſion will be obtained, if the bodies be not ſituated in a ſtraight line. As a direct inveſtigation, however, is always to be preferred to concluſions drawn from induction, it may be thought proper, before we apply any of the foregoing principles to the caſe of the action of bodies upon each other by impact, to ſhew how ſuch a direct inveſtigation to determine the point about which a body, having a motion communicated to it, begins to revolve, may be obtained; previous to which, however, ſome further conſiderations are neceſſary.

PROP. IX.

If a force acts upon a body in any given direction not paſſing through the center of gravity; to determine the plane of rotation, the direction in which the center of gravity begins to move, and its motion after.

Conceive a plane AYBZ (fig. 6.) to be ſupported upon a line AB paſſing through its center of gravity G, and ſuppoſe a force to act at any point D in that line, and in a direction perpendicular to the plane; then it is manifeſt, that ſuch a force can give the plane no rotatory motion about AB. Imagine now the ſupport to be taken away whilſt the force is acting at D, then it is evident, that as the plane had no tendency to move about AB as an axis, and the taking away of the ſupport can give it no ſuch motion, it will, by Cor. 2. Prop. VIII. begin its progreſſive motion in the direction in which the force acts; and as the force is ſuppoſed not to act at the center of gravity, it muſt at the ſame time have a rotatory motion about ſome axis, which, as it has no motion about AB, muſt lie ſomewhere in the plane, and perpendicular to AB; and conſequently *in ipſo motûs initio* the plane of rotation muſt be perpendicular to the plane AYBZ.

AYBZ. Let LCM, perpendicular to AB, be the axis about which the plane begins to revolve, and p, q be two equal particles of the plane ſimilarly ſituated in reſpect to AB, alſo qb, pa perpendicular to LCM. Now the centrifugal force of p, or its force in the direction ap is $p \times ap$, and that of q in the direction bq is $q \times bq$; to determine now how theſe forces will affect the motion of the plane, we may obſerve in the firſt place, that the force $p \times ap$, acting at a in the plane, muſt tend to give it a motion about an axis perpendicular to the plane; but as an equal force $q \times qb$ acts at q to give it a motion in a contrary direction, it is evident that the two forces will deſtroy each other, ſo far as they tend to generate any motion in the plane about an axis perpendicular to it; and hence it is manifeſt, that if the parts of the plane AYB, AZB, be ſimilar, and ſimilarly ſituated in reſpect to AB, the plane, after the commencement of the motion, will have no tendency to revolve about an axis perpendicular to it. Alſo, as the centrifugal force of each particle acts in a direction parallel to AB, it can give the plane no tendency to revolve about that line as an axis, and conſequently the plane of rotation will be preſerved as *in ipſo motûs initio*. Conceiving therefore the plane on each ſide the line AB to be ſimilar, and ſimilarly ſituated, ſuppoſe another plane to be fixed upon this, whoſe parts

on each ſide AB are ſimilar, and ſimilarly ſituated, and the force to act as before, then it is manifeſt, that as each plane endeavours to preſerve the ſame plane of rotation, the two planes connected will alſo continue to move in the ſame plane of rotation, for the action of one plane on another, on each ſide the plane of rotation, being equal, cannot tend to diſturb the motion in that plane; and as this muſt be true for any number of planes thus ſimilar and ſimilarly ſituated, it is evident, that if a force ſhould act upon a body, and each ſection, perpendicular to the direction of the force, ſhould be ſimilar on each ſide the plane paſſing through the direction of the force, and the center of gravity of the body, that that plane would be the plane of rotation in which the body would both begin and continue its motion. It appears alſo from what has been proved, that if every ſection on each ſide that plane had not been ſimilar, the plane of rotation would not *neceſſarily* have continued the ſame after the commencement of the motion. Hence all bodies, formed by the revolution of any plane figure, will have the axis about which they were generated, a fixt axis of rotation; to determine, however, every other axis of a body about which it would continue to revolve, would be foreign to the ſubject of this paper. Suppoſing therefore the plane of rotation to continue the

ſame

ſame (for in this paper I mean to confine my enquiries to ſuch caſes) imagine all the particles of the body to be referred to that plane orthographically, which ſuppoſition not affecting the angular motion of the body, the centrifugal force of all the particles, to cauſe the body to revolve about an axis perpendicular to that plane, will remain unaltered. Let LMNO (fig. 7.) be that plane, and ſuppoſe a force to act at A in the direction PA lying in the ſame plane, which produce until it meets LN, paſſing through the center of gravity G, perpendicularly in D; then by Cor. 2. Prop. VIII. the center of gravity G will begin its motion in a line parallel to PA, or perpendicular to LN; and conſequently the center C, about which the body begins to revolve, muſt lie ſomewhere in the line LN. Now the centrifugal force of any particle p is $p \times pc$; let fall pa perpendicular to LN, then the effect of that force at C, in a direction perpendicular to LN, will be $p \times pa$, and in the direction CL it will be $p \times ca$; but as the ſum of all the quantities $p \times pa = 0$, and the ſum of all the quanties $p \times ca$ = the body multiplied into CG, it follows from the ſame reaſoning as in Prop. III. that the point G will continue to move in a direction perpendicular to LN; and alſo, as the forces $p \times ca$ act in a direction perpendicular to that in which the center of gravity moves, its motion muſt be continued.

nued uniform. In the following Propoſitions, therefore, we ſuppoſe the axis of the body, after the commencement of the motion, to continue perpendicular to the plane paſſing through the direction of the force, and the center of gravity of the body, and that the body itſelf is orthographically projected upon that plane; alſo in the caſe of the action of two bodies on each other, the plane paſſing through the direction of the ſtriking body and point of percuſſion is ſuppoſed to paſs through the centers of gravity of each body; that the axis of each body after it is ſtruck continues perpendicular to that plane, and that each body is reduced to it in the manner above deſcribed.

PROP. X.

To determine the point about which a body, when ſtruck, begins to revolve.

Let LMNO (fig. 7.) repreſent the body, G the center of gravity, and PA the direction of the force acting at A, which produce till it meets LN, paſſing through G, perpendicularly in the point D; draw *pb* perpendicular to *p*C, on which (produced if neceſſary) let fall the perpendicular D*w*; C being ſuppoſed the point about which the body begins to revolve, and which, from the laſt Propoſition, is ſomewhere in the line LN. Becauſe the body, in conſequence

ſequence of the force acting at D, begins to revolve about C, and conſequently if immediately after the beginning of the motion a force were applied at D equal to it, and in a contrary direction, the motion of the body would be deſtroyed, it is evident, that the efficacy of the body revolving about C, to turn the body about D, ſhould any obſtacle be oppoſed to its motion at that point, muſt be equal to nothing; for were it not, the body, when ſtopped at D, would ſtill have a rotatory motion about that point, and conſequently two equal and oppoſite forces applied at D would not deſtroy each others effects, which would be abſurd. Now the force of a particle p, in the direction pw, being $p \times pC$, its efficacy to turn the body about the point D is $p \times pC \times Dw$; but by ſim. triang. $Dw : Db :: aC : pC$, $\therefore Dw = \frac{Db \times aC}{pC}$, and conſequently the efficacy to turn the body about $D = p \times Db \times aC = p \times Ca \times DC - Cb = p \times Ca \times DC - p \times pC^2$; hence the ſum of all the $p \times Ca \times DC$ – the ſum of all the $p \times EC^2 = 0$, and conſequently $CD = \frac{\text{ſum of all the } p \times PC^2}{\text{ſum of all the } p \times Ca}$, therefore D is the center of percuſſion, the point of ſuſpenſion being at C.

Cor. From this and the preceding Propoſition it appears, that every thing which was proved in Prop. v. vi. vii. holds here alſo in the caſe of the action of one body on another.

PROP.

PROP. XI.

Let a body P (fig. 8.) *moving with the velocity* V, *ſtrike the body* Q *at reſt in the point* A, *and in a direction* AD *paſſing through the center of gravity of the ſtriking body; to determine the velocity of each body after the ſtroke, ſuppoſing them to be elaſtic.*

The ſolution of this Propoſition depending on the ſame principles as that of Prop. III. we ſhall have, putting v equal the velocity of the center of gravity G after the ſtroke, on ſuppoſition that the bodies were non-elaſtic (DGC being ſuppoſed perpendicular to AD, and C the point about which the body Q begins to revolve) $\overline{V \times P \times CD} = \frac{v \times P \times CD^2}{CG} + \frac{v \times \text{ſum of all the } p \times Cp^2}{CG}$, and conſequently $v = \frac{V \times P \times CD \times CG}{\text{ſum of all the } p \times pC^2 + P \times CD^2}$; but it is well known, that the ſum of all the $p \times pC^2 = CG \times CD \times Q$, and hence $v = \frac{V \times P \times CG}{Q \times CG + P \times DC}$, and therefore if the bodies be ſuppoſed elaſtic, we have $\frac{2P \times V \times CG}{Q \times CG + P \times DC}$ for the velocity of the center of gravity G after the ſtroke. Now to determine the velocity of P, we have $\frac{P \times V \times CD}{Q \times CG + P \times DC}$ equal its velocity after the ſtroke from ſingle impact, and conſequently $V - \frac{P \times V \times CD}{Q \times CG + P \times DC} = \frac{Q \times V \times CG}{Q \times CG + P \times DC}$ is the velocity loſt

loſt by P from ſimple impact; hence if the bodies be elaſtic, $\frac{2\times Q\times V\times CG}{Q\times GC+P\times DC}$ will be the velocity loſt by P if elaſtic, and conſequently the velocity of P after the ſtroke $= V-\frac{2\times Q\times V\times CG}{Q\times GC+P\times DC}=\frac{P\times DC-Q\times GC}{Q\times GC+P\times DC}\times V$.

Cor. 1. If the direction AD paſſes through G, then CG being equal to CD, we have $\frac{2PV}{Q+P}$ = Q's velocity, and $\frac{P-Q}{P+Q}\times V$ = P's velocity, which is well known from the common principles of elaſtic bodies.

Cor. 2. If $P\times DC=Q\times GC$, or $P:Q::GC:DC$, then will the body P be at reſt after the ſtroke.

Cor. 3. If Q were infinitely great, the velocity of P after the ſtroke would be $=-V$ as it ought, for P would then ſtrike againſt an immoveable obſtacle.

Cor. 4. Whatever motion Q gains from the action of P, it would loſe, if, inſtead of ſuppoſing P to ſtrike Q, Q were to move in an oppoſite direction, and ſtrike P at reſt with the ſame velocity with which P ſtruck Q; in ſuch caſe, therefore, the velocity of Q after the ſtroke would be $V-\frac{2P\times GC\times V}{Q\times GC+P\times DC}=\frac{\overline{Q-2P}\times GC+P\times DC}{Q\times GC+P\times DC}\times V$.

Cor. 5. Hence if P be infinitely great, or Q be ſuppoſed to ſtrike an immoveable object, its velocity after the ſtroke will be $=\frac{DC-2CG}{DC}\times V$: hence when $DC=2GC$, the body Q will have no progreſſive motion after the

ſtroke, but would in ſuch caſe, if P were immediately taken away, continue to revolve about a fixed axis. It may alſo be obſerved, that when DC is greater than 2GC, or the velocity of Q is poſitive, that, becauſe it is impoſſible for Q to continue its progreſſive motion, it is only to be underſtood, that if immediately after the impact the body P were removed, the body Q would then proceed with ſuch a velocity.

Cor. 6. Suppoſe the bodies to be non-elaſtic, and let M be the magnitude of a body placed at D, which, being acted upon by P, ſhall have the ſame velocity generated as was before generated in the point D of the body Q; then by the common rule for non-elaſtic bodies, the velocity of M after the ſtroke will be $\frac{P \times V}{P+M}$, and hence $\frac{P \times V}{P+M} = \frac{P \times V \times DC}{Q \times CG + P \times DC}$, conſequently $M = Q \times \frac{GC}{DC}$.

Cor. 7. If a given quantity of motion were communicated to *any* point of the body Q, the progreſſive motion of that body after the ſtroke would be the ſame. For ſuppoſe the magnitude of the body P to be diminiſhed *ſine limite*, and its velocity to be increaſed in the ſame ratio, then, becauſe $\frac{P \times V \times CD}{Q \times CG + P \times DC}$ (which is the velocity of P after the ſtroke, if the bodies be non-elaſtic) = (becauſe P is infinitely ſmall) $\frac{P \times V \times CD}{Q \times CG}$, the velocity of P after the ſtroke

ſtroke from ſimple impact is finite, conſequently its motion muſt be infinitely ſmall, and therefore P muſt have communicated all its motion to Q: now in this caſe the velocity of Q $(=\frac{P\times V\times CG}{Q\times CG+P\times CD})=\frac{P\times V}{Q}$, which quantity is independent of the place where the force acts; in the ſame manner it would appear if we had ſuppoſed the bodies elaſtic.

PROP. XII.

Suppoſing every thing given as in the laſt Propoſition, except that the direction AD *does not paſs through the center of gravity g of the ſtriking body; to determine the velocity of each body after the ſtroke.*

Let AD (fig. 9.) be produced to meet FgO paſſing through *g*, the center of gravity of the ſtriking body, perpendicularly in F, and ſuppoſe O to be the point of the body P which is not diſturbed by the action of P on Q: now it appears from Cor. 6. Prop. xi. that if both bodies were non-elaſtic, and a body equal to $Q\times\frac{CG}{CD}$ were placed at D, the velocity of that body, from the action of P, would be equal to the velocity of the point D of the body Q; for the ſame reaſon, therefore, it appears, that if, inſtead of ſuppoſing

P to ſtrike Q in the direction FA, a body equal to $P \times \frac{GO}{FO}$ were to ſtrike Q at the ſame point and in the ſame direction (which direction is ſuppoſed to paſs through the center of gravity of that body) the effect on Q would be the ſame; hence, if in the quantity $\frac{V \times P \times CD}{Q \times GC + P \times DC}$, which from the laſt Prop. expreſſes the velocity of the point D after the ſtroke, on ſuppoſition that the bodies are non-elaſtic, we ſubſtitute for P a body equal to $P \times \frac{GO}{FO}$, we ſhall have $\frac{V \times P \times DC \times gO}{Q \times GC \times FO + P \times gO \times DC}$ for the velocity of the point D from the action of P; and conſequently $\frac{2 \times V \times P \times GC \times gO}{Q \times GC \times FO + P \times gO \times DC}$ = the velocity of the center of gravity G of the body Q, after the ſtroke, if the bodies be perfectly elaſtic. To determine now the velocity of the ſtriking body, let *of*, perpendicular to *og*, be the ſpace deſcribed by the point O in the firſt inſtant of time after the ſtroke, which, as that point is not diſturbed by the action of the bodies on each other, may repreſent the velocity of P before the ſtroke, and let F*b* repreſent the velocity of the point F after the ſtroke; join *fb*, and draw *gd* perpendicular to *og*, and then will *gd* repreſent the velocity of the center of gravity *g* of the ſtriking body after the ſtroke. Draw *fc* perpendicular to FA, and produce *gd* to meet *fc* in *e*; now the velocity

loſt

loſt by P at the point F by ſimple impact being equal to $V - \frac{V \times P \times DC \times gO}{Q \times GC \times FO + P \times gO \times DC} = \frac{V \times Q \times GC \times FO}{Q \times GC \times FO + P \times gO \times DC}$, we ſhall have *bc* the velocity loſt by the point F, on ſuppoſition that the bodies are perfectly elaſtic (ſuppoſing *of* to repreſent the value of V) equal to $\frac{2 \times V \times Q \times GC \times FO}{Q \times GC \times FO + P \times gO \times DC}$, and therefore by ſim. triang. *fc* (FO) : *cb* : : *fe* (*og*) : *ed* = $\frac{2 \times V \times Q \times GC \times gO}{Q \times GC \times FO + P \times gO \times DC}$ = the velocity loſt by the center of gravity *g*, and hence $V - \frac{2 \times V \times Q \times GC \times gO}{Q \times GC \times FO + P \times gO \times DC} = \frac{V \times Q \times GC \times FO + V \times P \times gO \times DC - 2 \times V \times Q \times GC \times gO}{Q \times GC \times FO + P \times gO \times DC}$ = the velocity of P after the ſtroke. Now, as it appears from Prop. IX. that the progreſſive motion of a body, when left to move freely, continues uniform and in the ſame direction, it follows, that the expreſſions for the velocities of each body in the firſt inſtant after the ſtroke, both in this and the preceding Propoſition, will repreſent the uniform progreſſive velocities with which the bodies will continue to move, and conſequently the place of each body, at the end of any given time after impact, may eaſily be determined.

Cor. I. If the direction FA paſſes through *g*, then FO and *go* becoming infinite, we ſhall have $\frac{2 \times V \times P \times GC}{Q \times GC + P \times DC}$ for the velocity of Q, and $\frac{V \times P \times DC - V \times Q \times GC}{Q \times GC + P \times DC}$ for the velocity

velocity of P, agreeable to what was proved in the laſt Propoſition.

Cor. 2. Hence the point about which P begins its rotatory motion may eaſily be found; for produce (if neceſſary) *fb* and OF to meet in *a*, and *a* will be the point required; and by ſim. triang. $bc \left(= \frac{2 \times V \times Q \times GC \times FO}{Q \times GC \times FO + P \times gO \times DC}\right) : cf :: fo\,(= v) : oa = \frac{Q \times GC \times FO + P \times gO \times DC}{2 \times Q \times GC}$, and hence $Fa = \frac{P \times gO \times DC - Q \times GC \times OF}{2 \times Q \times GC}$.

Cor. 3. If, inſtead of ſuppoſing Q to have been at reſt, it had been moving forward in a direction parallel to that of the body P, with the velocity v, the motion of each body after the ſtroke may eaſily be determined: for conſidering P as acting upon Q with the velocity $V - v$, we ſhall have by this Propoſition (putting $2M = \frac{2P \times GC \times GO}{Q \times GC \times FO + P \times gO \times DC}$) $\overline{V - v} \times 2M =$ the velocity communicated to G, therefore $v + \overline{V - v} \times 2M =$ the velocity of Q after the ſtroke: alſo $\overline{V - v} \times M \times \frac{CD}{CG} =$ the velocity gained by the point D from ſimple impact, and conſequently the velocity of that point after $= v + \overline{V - v} \times M \times \frac{CD}{CG}$, hence $V - v - \overline{V - v} \times M \times \frac{CD}{CG} =$ the velocity loſt by P at the point F from ſimple impact, therefore P's velocity after the ſtroke $= V - \overline{V - v} - \overline{V - v} \times M \times \frac{\overline{CD}}{CG} \times \frac{2gO}{FO}$. In the ſame

Fig. 1.

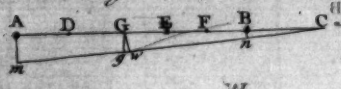


Fig. 4.

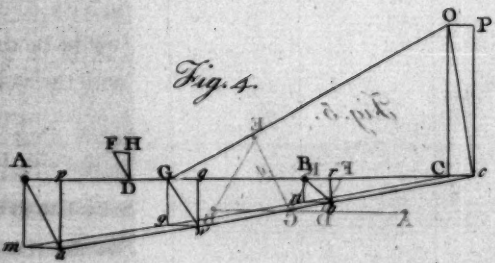


Fig. 7.

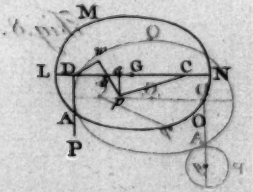
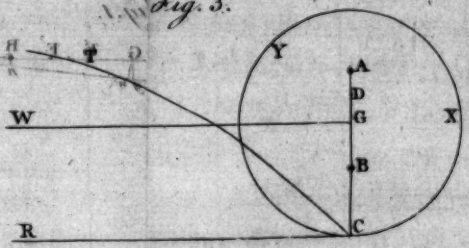


Fig. 2.

Fig. 3.



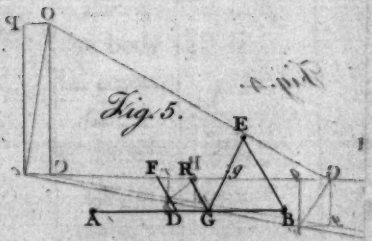
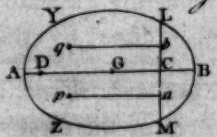


Fig. 5.

Fig. 6.



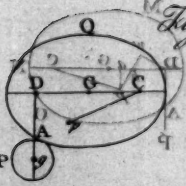


Fig. 8.

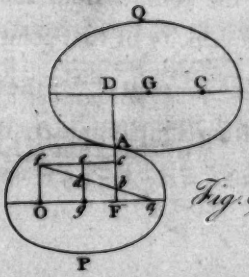


Fig. 9.

manner it might have been determined, had Q moved in an oppoſite direction.

Cor. 4. Hence alſo we may eaſily determine the motion of each body after the ſtroke, ſuppoſing Q had not been moving in a direction parallel to the motion of P, by reſolving Q's motion into two parts, one parallel to the motion of P, and the other perpendicular; and finding by the preceding what would be the effect of the parallel motions, and then compounding Q's motion, after the ſtroke from that conſideration, with the motion it had in a direction perpendicular thereto before the ſtroke.

Cor. 5. The point *a* of the body P will deſcribe (when that body after the ſtroke has any progreſſive motion) the common cycloid.

Cor. 6. Hence, therefore, the times of the revolutions of each body may be determined as in Prop. VI.

Cor. 7. If the bodies had any rotatory motion before impact, every thing relative to the motion of the bodies after the ſtroke might have been determined from the ſame principles.

XXXIII. *Continuation of the Caſe of* James Jones. *By* Richard Browne Cheſton, *Surgeon to the* Glouceſter *Infirmary. Communicated in a Letter to Mr.* Henry Watſon, *Surgeon to the* Weſtminſter *Hoſpital.* *(See p.* 323.)

Read November 16, 1780.

DEAR SIR,

I HAVE it at laſt in my power to inform you of the ſtate the bones of the *pelvis* appeared in after a *maceration* of five months: for though by very ſeldom changing the water, and keeping the veſſel containing it rather in a warm place, I ſuffered the higheſt putrefaction to come on, it took up that ſpace of time before the ſoft parts were entirely deſtroyed.

In my account of the ſtate in which I found the *thoracic duct* of the ſame ſubject, I mentioned my ſuſpicions (from the only examination I then had it in my power to make) that a conſiderable part of the ſubſtance of the *os innominatum* was deſtroyed or abſorbed; but *maceration* has cleared up this circumſtance, and ſhewn, that the depth the probe entered, and the gritty reſiſtance

reſiſtance I felt in the body of the tumor, was not from its paſſing through a carious or diſeaſed part, as there was reaſon then to ſuppoſe, but from the quantity of oſſeous matter depoſited on the outward ſide of the *os innominatum*; and that the part ſo loaded with it externally, and as it afterwards proved to be *internally*, was apparently in a ſound ſtate.

It is difficult to fix upon the preciſe part where the tumor occupying the right moiety of the *pelvis*, and containg the boney matter, originated. There is, however, every reaſon to ſuppoſe it began low down in the *pelvis* under the *peritoneum*, as the ſurface of the tumor was evidently covered with that membrane; and from every circumſtance in the appearance of the tumor, as well as a careful examination of it, during the progreſs of its diſſolution, it is probable, that paſſing out of the *pelvis* anteriorly through the *foramen ovale*, as well as laterally under the *iſchium*, and riſing upwards over the ſpine of the *ilium*, it formed a complete communication from within outwardly, ſo as to envelope the whole of the right *os innominatum* entirely.

As the ſoft parts of the tumor decayed in maceration, great quantities of boney matter in irregular forms and of different ſizes were found in the water at the bottom of the pan; and as no force, nor even

motion, had been uſed which could have ſeparated this matter from what remained adherent to the bone, it is highly probable it was ever depoſited in, and diſperſed through, the tumor in a detached ſtate.

The tumor, externally, bore the uſual appearances of a diſeaſed or enlarged gland; but, by degrees) as I had opportunities to obſerve upon changing the water) the whole appearance was changed, and the boney matter, as the maceration proceeded, ſeemed ſurrounded by a hard, white, and rather tranſparent ſubſtance, not much unlike ſuet, in which ſtate it principally reſiſted the diſſolvent power of the water.

When the bone in general ſeemed ſufficiently cleanſed for drying, I found in one part ſome remains of this ſuety ſubſtance; but as I was unwilling to continue the whole in water any longer for the ſake of this ſmall portion, and thought beſides that in its drying I might get ſome inſight into its nature, I expoſed it to the open air, and was ſurprized to find that in the courſe of three days it was intirely diſſipated, ſcarcely a trace of it remaining, unleſs that, in the particular portion alluded to, the boney matter was of a more duſky colour than elſewhere.

I had now the ſatisfaction to find the attention and care beſtowed upon the bones during the maceration amply repaid by the ſingularities which they preſented.

The left *os innominatum* was perfectly free from any unnatural appearance in every part, even to its junction at the *ſymphyſis* of the *pubis*; but there the line was drawn, and diſeaſe immediately began to ſhew itſelf through the whole of the right *os innominatum*, and to advance as it were from a ſuperficial ulceration to excreſcences in the greateſt quantity.

It is remarkable, that the cartilage connecting the *oſſa pubis* ſhould be ſo complete a boundary to the diſeaſe; for though the external *lamella* was in all that part of the *os pubis* and *iſchium* (particularly at the *ramus* of the latter) which united forms the *foramen ovale*, not the leaſt deficiency is to be obſerved in the left *os pubis*. So intereſting to the knowledge of the nature of this diſeaſe is it to obſerve that the extent of the tumor, which terminated exactly at this part, ſhould likewiſe as exactly have limited its effects or conſequences.

So far then as the external *lamella* of the *os pubis* and *iſchium* was deficient, ſo far theſe bones preſented that roughneſs and irregular loſs of ſubſtances which is commonly denominated ulceration or ſuperficial *caries*. The bottom of the *acetabulum* had likewiſe ſuffered in a ſimilar manner: but over the whole of the *ilium*, both externally and internally, the *lamella* ſeemed very little injured, though covered by vaſt quantities of boney

matter branching out into various forms of different sizes, which a minute examination and careful attempt to separate ascertained to be mere, though firm, adhesions to the surface of the bone, and adventitious to the part on which they were found.

I am, &c.

Gloucester,
November 1, 1780.

EXPLANATION OF THE DRAWINGS.

N° I. A front view of the *pelvis*.

It may only be necessary to observe here, as the drawing is so expressive of the original, that the centre of the external part of the *ilium* is not covered with boney matter in such quantities, or of such a size, as there is around and over the posterior part of the *crista* or spine, and where this bone forms a junction with the *os sacrum*; yet there is not the space of a six pence which has not more or less of it on it.

The *os sacrum*, though apparently found, is likewise sprinkled with it in several places.

N°

No. 1.

N. 2.

N° II. A ſide view of the *pelvis*, ſhewing,

1. In the *acetabulum* a ſlight eroſion.

2. The boney matter in different forms and ſizes on the outward ſide of the *ilium*.

3. The *os ſacrum* ſlightly expreſſed, reſting on a cork.

4. The *criſta* of the *ilium* at the poſterior part thickly covered with boney matter; at the anterior, free from it.

I have made no references in the drawings, as the above remarks will be ſufficiently obvious.

THERMOMETRICAL EXPERIMENTS

AND

OBSERVATIONS.

By TIBERIUS CAVALLO, F.R.S.

Who was nominated by the President and Council to profecute Diſcoveries in Natural Hiſtory, purſuant to the Will of the late HENRY BAKER, Eſq. F. R. S.

Read at the ROYAL SOCIETY, June 15, 1780.

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THERMOMETRICAL EXPERIMENTS, &c.

HAVING been appointed by the Preſident and Council of the Royal Society to write the annual diſſertation, purſuant to the inſtitution of HENRY BAKER, Eſq. F. R. S. I determined to preſent to the Society the following account of ſome thermometrical experiments and obſervations, the greateſt part of which were made as long ago as the year 1776. The courſe of thoſe experiments having been often interrupted by mechanical defects in the conſtruction of inſtruments, and by various other circumſtances, muſt not be conſidered as complete inveſtigations of the propoſed views, but rather as firſt attempts, which are to be proſecuted in caſe of better opportunities. They contain a few facts, which to me ſeem new and worthy of notice; and, perhaps, the obſervations concerning the conſtruction and uſe of the inſtruments uſeful in experiments of this ſort, may be of uſe to thoſe perſons who have the opportunity, and are willing, to proſecute this experimental inquiry.

Having read in a volume of the Philoſophical Tranſactions the account of an experiment made with a thermometer, whoſe bulb was painted black, and was expoſed to the rays of the Sun, in which caſe it had been found, that the thermometer ſhewed a much greater degree of heat than when not blackened, I was deſirous of trying the ultimate limits of this difference. For which purpoſe I conſtructed two thermometers, the ſcales of which (being made by trial) coincided ſo perfectly well together, that when the thermometers were put in equal circumſtances, no difference could be perceived between the degrees of heat ſhewn by them. The method uſed to graduate thoſe thermometers is as follows. The ſcale of one of the thermometers was made in the uſual manner, *viz.* by finding the boiling and freezing water points, and dividing that diſtance into 180 equal parts, which are the degrees according to FAHRENHEIT. Here no regard was paid to the barometrical height at the time of finding the boiling water point, it being uſeleſs for the experiment for which the inſtrument was intended. This done, the balls of the two thermometers were put into hot water, and according as the water cooled, and conſequently the mercury deſcended in the tubes, different marks were put upon the tube of the ungraduated thermometer,

which were nominated after the degree of heat ſhewn at the ſame time by the graduated thermometer. Thus, for inſtance, a mark was put at 140°, another mark was put at 135°, another at 130°, and ſo on. It is plain, that in thoſe points, the thermometers when put in equal circumſtances, muſt coincide perfectly together. Now, on making the ſcale thoſe points are marked firſt, and as the diſtances between thoſe marks were very ſmall (conſiſting of a few degrees) they were divided with the compaſſes in the proper number of equal parts or degrees; and in this manner the ſcale of the other thermometer, was completed, by which means, although theſe two thermometers did not coincide ſo well with other thermometers, yet they coincided perfectly well together, as muſt inevitably be the caſe, even upon the ſuppoſition that their tubes were not perfectly cylindrical. The length of a degree on the ſcale of thoſe thermometers was a little more than $\frac{1}{10}$th of an inch, and although thoſe ſcales were divided into degrees only, yet by inſpection a perſon a little verſed in theſe obſervations could eaſily diſtinguiſh the height of the quickſilver within a quarter of a degree.

Theſe thermometers were both fixed upon the ſame frame at the diſtance of about one inch from one another, having the balls quite detached from the frame, and in this manner they were expoſed to the Sun, or to the light of a lamp.

When these thermometers were exposed to the Sun, or kept in the shade, they shewed the same degree precisely. The difference between the degree shewn by these thermometers when exposed to the Sun, and when kept in the shade at about the same time of the day, was very trifling.

When the ball of one of those thermometers, which we shall call A, was painted black with Indian ink, or with the smoke of a candle, and that of the other thermometer B was left clean, on being exposed to the Sun they shewed different degrees of temperature; the quicksilver in the tube of A was much above the quicksilver in the tube of B. This difference sometimes amounted to about 10°. but it was never constant, varying according to the clearness of the Sun's light as well as of the air, and also according to the different degrees of temperature of the atmosphere.

Keeping the frame with those thermometers, one of which had the ball painted black, hung on the side of a window, I observed a remarkable fact, *viz.* that these thermometers shewed unequal degrees of heat, not only when presented to the Sun, but also when exposed to the strong day-light. I cleaned the bulb of the thermometer A, and blackened that of B, but the effect was constant, *viz.* the quicksilver in the tube of the thermometer, whose

whoſe bulb was painted black, was conſtantly higher than the other, whenever they were expoſed to the ſtrong day-light. This difference was commonly about one-third of a degree, but ſometimes it amounted to three-fourths, and even to a whole degree. The ſituation in which thoſe thermometers were uſually placed was ſuch that the light of the Sun could not be reflected upon them by any object ſtanding before; but the experiment anſwered even when the Sun was hidden by clouds.

This obſervation ſeemed to ſhew that, perhaps, every degree of light is attended with a proportionate degree of heat; and induced me to try, in a ſimilar manner, whether, by directing the concentrated light of the Moon upon the blackened ball of one of theſe thermometers, I could render ſenſible the effect of that light [a]. But although I attempted it ſome time ago with a large lens ſeveral times, and have lately tried it again with a burning mirror of eighteen inches diameter, yet ſometimes for want of proper means of obſerving the height of the mercury in the tubes of the thermometers, ſometimes for want of a continued clear light of the Moon, and in ſhort from one unfavourable circumſtance or

(a) The concentrated light of the Moon has often been thrown upon thermometers without any effect; but I do not know that any blackened thermometer was ever uſed before for this purpoſe.

other,

other, I have not yet been able to make a fair and decisive trial of this experiment.

The light of the Sun being very inconstant on account of clouds and of its diurnal motion, I thought to make some experiments with the above mentioned two thermometers, by exposing them to the light of a lamp, and I found that this light had a considerable effect upon them.

The ball of one of the thermometers being blackened, and both being set at two inches distance from the flame of a lamp, they both rose from 58°, at which the mercury stood before the lighting of the lamp, to 65°$\frac{1}{2}$, and the blackened thermometer to 67°$\frac{1}{2}$. Another time, being set at the same distance from the lamp, the uncoloured thermometer came up to 67°$\frac{3}{4}$, and the blackened one to 68°$\frac{3}{4}$. In short, by various repeated trials it appeared, that the difference generally amounted to about 1°. When the thermometers were put farther than two inches from the lamp, this difference decreased, and at about fourteen or fifteen inches it vanished quite.

It is mathematically true, that emanations which proceed from a center, and expand in a sphere, must continually become more and more rare in proportion to the squares of the distances from the center. Thus it is said, that the intensity of light proceeding from a luminous body

body at the double, treble, quadruple, &c. of a given diſtance from that body, muſt be reſpectively four, nine, ſixteen times leſs denſe. The ſame thing may be ſaid of heat.

Being willing to aſcertain this truth by actual experiment, I placed ſeveral thermometers, whoſe balls were not painted, at different diſtances from the flame of the lamp, and expected to find, when the thermometer at four inches diſtance was 1° above that placed at eight inches diſtance, the thermometer placed at two inches diſtance ſhould be 4° higher. But upon trying this experiment various times, placing the thermometers at different diſtances from the flame of the lamp, and making the proper calculations agreeable to thoſe diſtances, it appeared, that the intenſity of the heat did not decreaſe exactly in the duplicate proportion of the diſtances from the flame of the lamp, but ſhewed a very odd irregularity. It ſeemed to decreaſe faſter than the duplicate proportion of the diſtances for the ſpace of two inches and a half or three inches, after which it decreaſed much ſlower. Whether this effect may be attributed to ſome different ſtate of the air's purity at different diſtances from the flame of the lamp, or to the vapours proceeding from the flame, I cannot take upon me to determine.

The

The above mentioned experiments gradually induced me to try the effect of the light of the Sun and of a lamp upon thermometers whose balls were painted with different colours. Dr. FRANKLIN's experiment with the pieces of cloth set upon snow that was exposed to the Sun is very well known. The doctor found, that those pieces of cloth, whose colour was darker, sunk deeper in the snow than the others, by which it appears, that they became hotter. My view was to examine those different degrees of heat imbibed by different coloured substances with precision, in order to observe if they kept any proportion to the spaces occupied by the prismatic colours in the prismatic spectrum, or if they followed any other discoverable law; but those attempts met with many difficulties, the greatest of which was the choice of colours. The water colours that are commonly used, as carmine, sap-green, &c. are of so different a nature from one another, that when the balls of the thermometers were painted with them, their surfaces were not equally smooth, which occasioned great difference in the effect; for I found, that two thermometers, whose balls had been painted with the same colour, but the paint laid smoother on one than on the other, shewed different degrees of heat when they were both exposed to the rays of the Sun.

I attempted

I attempted to make thermometers with tubes of differently coloured glass, but when a ball was formed with any of those tubes, the substance of the glass in the ball, being much thinner than in the tube, differed very little from clear colourless glass.

To include the thermometers in close boxes, in which the rays entered through coloured glasses, was also found ineffectual; not only because the colours so transmitted were far from being homogeneous, but especially because some of those glasses are much more opaque than others, even of the same colour.

The least ambiguous method, therefore, was that of painting the balls of the thermometers with water colours, taking care to lay them as equally smooth as possible. In this manner I repeated several experiments, using sometimes a dozen of thermometers at once, whose balls were painted with various colours, and were exposed to the Sun; and from a vast number of experiments, and after some weeks observation, it could be only deduced, that if the colours, with which the balls of the thermometer were painted, were pretty like the prismatic colours, those thermometers shewed a greater degree of heat, whose colours were nearer to the violet in the order of the prismatic colours, and contrarywise; but they were all, even that painted with white lead, in some

intermediate degree between the blackened thermometer and the naked or unpainted one. If the colours had not the proper degree of denſity, the effects were very different: thus a thermometer painted with a light blue was lower than another thermometer painted red with good carmine.

I ſhall now deſcribe the manner of conſtructing the ſcales of thoſe thermometers, which was contrived ſo as to be very expeditious; becauſe ſome of thoſe thermometers were often broken by ſome accident or other, and that new ſchemes often required new thermometers, to conſtruct the ſcales of which in a formal manner would have required a very long time. Thoſe methods therefore may be of uſe to other perſons.

When the thermometers were intended to be expoſed to the flame of the lamp, at a given diſtance from it, their ſcales were drawn upon ſlips of paper which were glued to their tubes in the manner repreſented by fig. 1. The thermometers were then ſet horizontally upon a book, ſo that their balls were out of the book, and at any required diſtances from the flame of the lamp, which diſtances were meaſured with a pair of compaſſes. But when the thermometers were only to be expoſed to the Sun, I then uſed the following very expeditious method, ſee fig. 2. Upon an oblong board ABCD of about 14 inches by 18,

and nearly one inch thick, I paſted a piece of white paper, and delineated upon it a right-angled triangle EGF, one ſide GF of which came very near the edge CD of the board; the other ſide EF, which ſtands perpendicular to GF, was divided into equal parts, repreſenting degrees of FAHRENHEIT's thermometrical ſcale. The loweſt of thoſe degrees was near the freezing point, and the upper was not above the 110°, it being as much as I was in want of. From the point G right lines were drawn to all the degrees in the ſcale EF, and many other right lines parallel to the ſcale EF, and conſequently perpendicular to the baſe GF, were drawn through the whole area of the triangle. Now when the thermometers were conſtructed, I found, by compariſon with a ſtandard thermometer, two points whatever, which I marked with the file upon the tube, and by a note of theſe thermometers I knew to what degrees they anſwered. Thus, for inſtance, upon the thermometer HL two marks were made, *viz.* I anſwering to the 40th degree, and K to the 70th degree. Now when this thermometer was to be uſed I placed it upon the board ABCD with the tube parallel to the ſcale EF, which could be eaſily done by the help of the parallel lines drawn upon the triangle, the ball of the thermometer being out of the board; thus I ſlid the thermometer backwards and forwards till the mark I,

which was the 40th degree, coincided with the line, which went from the point G to the 40th degree on the ſcale, and the mark K coincided with the line which went from G to the 70th degree. In that place the thermometer was left, and in order to keep it ſteady, a piece of lead (O) was uſually put upon the extremity of its tube. In this ſituation the interſections of the tube, and the lines drawn from G to the degrees in EF, ſhewed the degrees, or ſerved for a ſcale to the thermometer HL. Thus, ſuppoſe that the quickſilver in its tube was at P, it is plain, that it was at 55°, becauſe at that point the tube is interſected by the line which goes from G to the 55th degree on the ſcale EF. This board, therefore, ſerved for a univerſal ſcale, and upon it I uſed to fix ſeveral thermometers at a time, and expoſe them all together to the Sun.

I ſhall conclude this paper with mentioning an experiment, which, although not thermometrical, is yet uſeful in removing a wrong notion ſome perſons have concerning the effect of light.

Having ſeen in ſome book that the common black pyrophorus, or HOMBERG's pyrophorus, was impaired by light, I was deſirous to try the truth of this aſſertion. Accordingly towards the beginning of the laſt year I prepared ſome pyrophorus, and incloſed portions of it in three glaſs tubes, which were immediately ſealed

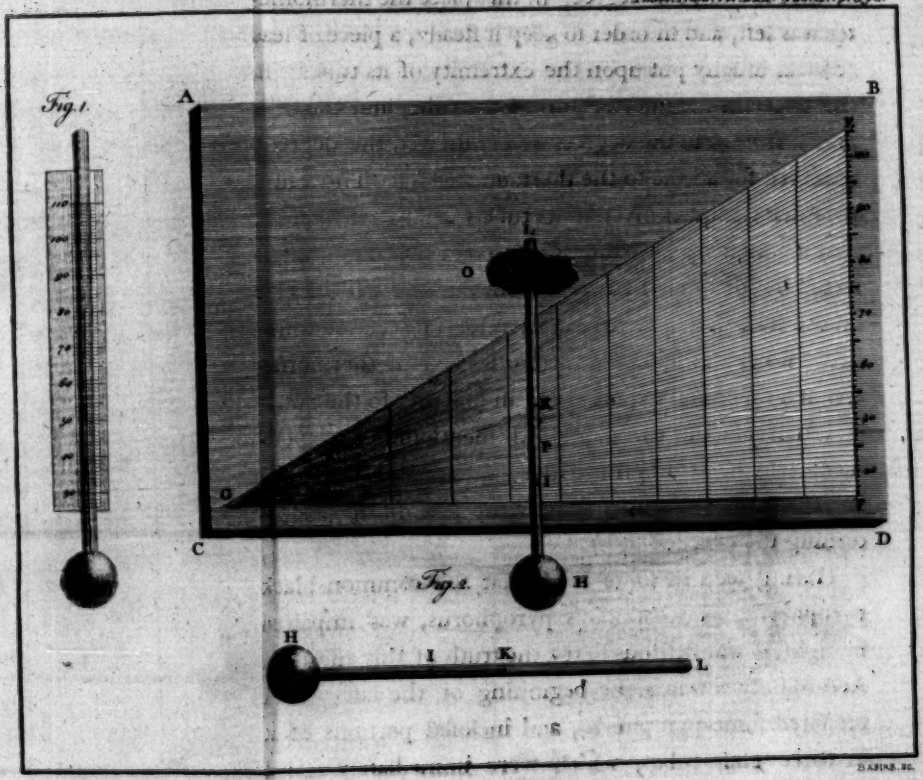
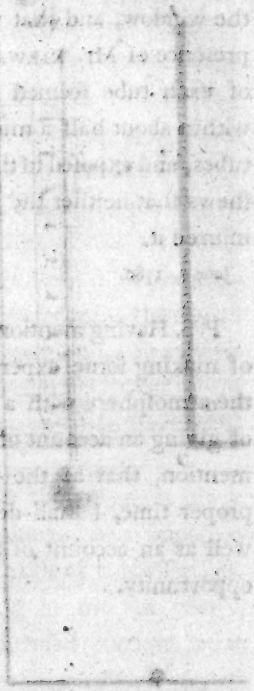


Fig. 1.
A
B
O
C
D
Fig. 2.
H
H
I
K
L



hermetically, and on the 20th of May, 1779, two of them were ſuſpended to a nail out of a window, and the third was wrapped up in paper, and was incloſed in a box, where not the leaſt glimmering of light could enter. In this ſituation they were left for above a year, and laſt week I broke one of thoſe that had been kept out of the window, and that which had been in the dark, in preſence of Mr. KIRWAN, F. R. S.; but the pyrophorus of each tube ſeemed to be equally good, taking fire within about half a minute after being taken out of the tubes, and expoſed to the air upon pieces of paper, which ſhews that neither the preſence or abſence of light had injured it.

June 7, 1780.

P. S. Having mentioned to ſeveral perſons my intention of making ſome experiments upon the temperature of the atmoſphere with a new metalline thermometer, and of giving an account of them in this paper, I muſt here mention, that as the inſtrument was not finiſhed in proper time, I ſhall defer giving a deſcription of it, as well as an account of the experiments, &c. to another opportunity.

P. S. Having mentioned several particulars in my journal of making some experiments upon the temperature of the atmosphere with a new metalline thermometer, and of giving an account of them in this paper, I must here mention, that as the instrument was not finished in proper time, I shall defer giving a description of it, as well as an account of the experiments, &c. to another opportunity.

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MDCCLXXX.

ADDENDA.

At the end of page 386 *add,*

In fig. 4. draw a circle through the points N, M, T; and at the maximum where $TP^2 = PM \times NP$ this circle will touch CA produced in T. From E the center of this circle draw EF perpendicular to NM, alſo the radii EN and EM; and FN is the ſine of NEF, or half NEM, or of its equal MTN, to the radius EN. But $EN = ET = FP = \frac{FN+FM}{2}$, and $FN = \frac{FN-FM}{2}$. Therefore $FN + PM$ is to $FN - FM$, or $CD + CB$ is to $CD - CB$, or $CA + CB$ is to $CA - CB$, as radius is to the ſine of the greateſt angle of deviation, which is therefore equal to $\frac{CA-CB}{CA+CB}$, radius being unity.

ERRATA to VOL. LXX.

Page 6, *line antepenult. read* be nearly mathematically.
6, *l. penult. dele* yet.
7, *l.* 13, *dele* ſection ABC, or
7, *l.* 18, *at the end of the line add* very nearly.
394, *l.* 15, *tranſpoſe* general equation *to the beginning of the line above.*
402, *l.* 6, 7, 8, *for* 9143 *r.* 9443.
405, *l.* 7, *for the laſt,* $1+\sqrt{-3}$ *r.* $1-\sqrt{-3}$.
405, *l.* 11, *for the laſt* $-\frac{\sqrt{-3}}{2}$ *r.* $+\frac{\sqrt{-3}}{2}$.
443, *end of the 1ſt line, for* and x *r.* and X.
548, *l.* 10, *for* circumſtances *r.* and which are.

*** There are FIFTEEN Plates in this Volume.